Chapter 3

3-6

(a.) $f_s = 2B = 2(100) = \underline{\underline{200 \text{ samples/sec}}}$

(b.) Using the results given in prob. 3-7.

$$n \geq 3.32 \log_{10}\left(\frac{50}{P}\right) = 3.32 \log_{10}\left(\frac{50}{0.1}\right)$$

$$= 8.96$$

$\underline{\underline{n = 9 \text{ bits/word}}}$

(c.) $R = \left(\frac{n \text{ bits}}{\text{word}}\right)\left(\frac{f_s \text{ words}}{\text{sec}}\right) = 200(9) = \underline{\underline{1.8 \frac{\text{k bits}}{\text{sec}}}}$

(d.) For binary PCM $D = R$

eq. (3-74) $D = \frac{2B}{1+r}$, for B_{min}, $r = 0$

$\Rightarrow \quad B = \frac{D}{2} = \underline{\underline{900 \text{ Hz}}}$

3-8

(a.) $f_s \geq 2\, B_{analog} = 2(20 \text{ kHz}) = 40 \frac{\text{k samples}}{\text{sec}}$

For 8X oversampling of the recovered PCM signal
(used to increase f_s 8X and simplify LPF requirements)

$\ni f_{8x} = 8 f_s = 320 \text{ k samples/sec}$

$B_{null} = R = n f_{8x} = \left(16 \frac{\text{bits}}{\text{sample}}\right)\left(320 \frac{\text{k samples}}{\text{sec}}\right) = \underline{\underline{5.12 \text{ MHz}}}$

(b) Using (3-18)

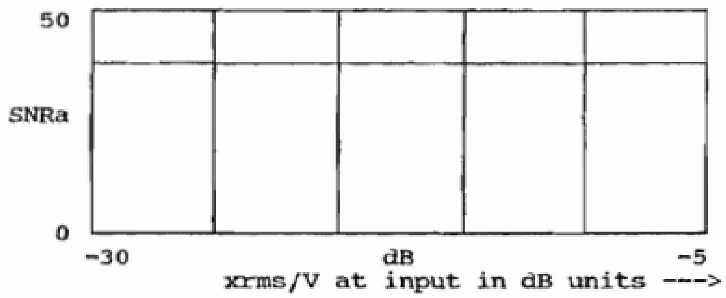
$\left(\frac{S}{N}\right)_{peak} = 6.02n + 4.77 \text{dB} = 6.02(15) + 4.77 = \underline{\underline{94.77 \text{dB}}}$

3-14

dB := −30,−29 ..−5   M := 256 $n := \frac{\log(M)}{\log(2)}$

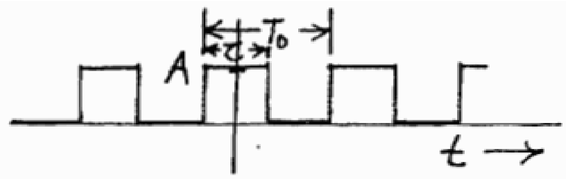
μ := 255   n = 8

SNRa := 6.02 n + 4.77 − 20 log(ln(1 + μ))



3-17

For alternating data the waveform is periodic where $T_0 = 2T_b$.



From (2-109) the spectrum is

$$W(f) = \sum_{-\infty}^{\infty} c_n\, \delta(f - n f_0)$$

Where
$$c_n = \frac{1}{T_0}\int_a^{a+T_0} w(t)\, e^{-jn\omega_0 t}\,dt = \frac{1}{T_0}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A\, e^{-jn\omega_0 t}\,dt$$

$$= \frac{A}{T_0}\,\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Bigg|_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = \frac{2A}{T_0}\underbrace{\left(\frac{e^{-jn\omega_0\tau/2} + e^{+jn\omega_0\tau/2}}{-2jn\omega_0}\right)}_{\sin(n\omega_0\tau/2)}$$

$$\Rightarrow c_n = \frac{2A\tau}{T_0 2}\,\frac{\sin(\omega_0 n\tau/2)}{\frac{n\omega_0\tau}{2}} = \frac{A}{2}\left(\frac{\tau}{T_b}\right)\frac{\sin\left(\frac{n\pi}{2}\frac{\tau}{T_b}\right)}{\left(\frac{n\pi}{2}\frac{\tau}{T_b}\right)}$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T_b}$$

$$\Rightarrow W(f) = \sum_{n=-\infty}^{\infty}\frac{A}{2}\left(\frac{\tau}{T_b}\right)\left(\frac{\sin\left(\frac{n\pi}{2}\frac{\tau}{T_b}\right)}{\left(\frac{n\pi}{2}\frac{\tau}{T_b}\right)}\right)\delta\left(f - \frac{n}{2}R\right) \quad (A)$$

where $R = \frac{1}{T_b}$ = bit rate

3-17 (Continued)

(a) Using (A) for NRZ signaling with $\tau = T_b$

$$|W(f)| = \sum_{-\infty}^{\infty} \frac{A}{2} \left| \frac{\sin\left(\frac{n\pi}{2}\right)}{\left(\frac{n\pi}{2}\right)} \right| \delta\left(f - \frac{n}{2}R\right)$$

Unipolar NRZ (alternating data)

If the data are a sequence of four "1"'s followed by four "0"'s, the waveform would have the same shape except T_0 would be 4 times as large. i.e. $T_0 = 8T_b$.

$$\Rightarrow |W(f)| = \sum_{-\infty}^{\infty} \frac{A}{2} \left| \frac{\sin\left(\frac{n\pi}{2}\right)}{\left(\frac{n\pi}{2}\right)} \right| \delta\left(f - \frac{n}{8}R\right)$$

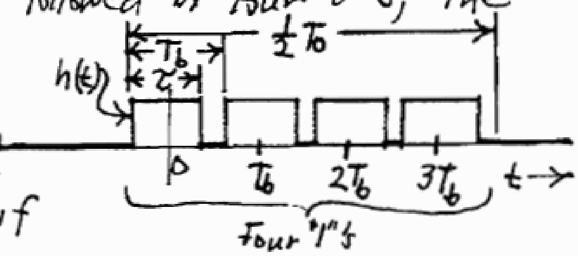
Unipolar NRZ 4 "1"'s & 4 "0"'s

(b) Using (A) for RZ signaling with $\tau = \frac{3}{4}T_b$,

$$|W(f)| = \sum_{-\infty}^{\infty} \frac{3}{8}A \left| \frac{\sin\left(\frac{3}{8}n\pi\right)}{\left(\frac{3}{8}n\pi\right)} \right| \delta\left(f - \frac{n}{2}R\right)$$

Unipolar RZ (alternating data)

For RZ with four "1"'s followed by four "0"'s, the periodic waveform would appear as shown where $T_0 = 8T_b$. The mathematical calculations are simplified if (2-112) is used



$$c_n = f_0\, H(nf_0)$$

where $h(t)$ is the basic waveform that is repeated to create the periodic waveform (as shown in the figure). $h(t)$ consists of the superposition of four rectangular pulses. Using the time delay theorem of Table 2-1

3-17 (Continued)

and the rectangular pulse spectrum of Table 2-2

$$H(f) = A\tau \frac{\sin(\pi f \tau)}{\pi f \tau}\left[1 + e^{-j\omega T_b} + e^{-j\omega 2T_b} + e^{-j\omega 3T_b}\right]$$

or

$$c_n = \frac{A\tau}{8T_b} \frac{\sin\left(\frac{n\pi}{8}\frac{\tau}{T_b}\right)}{\left(\frac{n\pi}{8}\frac{\tau}{T_b}\right)}\left[1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{2}} + e^{-j\frac{3}{4}n\pi}\right]$$

$$f = nf_0 = \frac{n}{T_0} = \frac{n}{8T_b}$$

For RZ with $\tau = \frac{3}{4}T_b$, this becomes

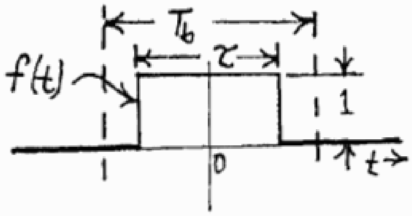
$$c_n = \frac{3}{32} A\left(\frac{\sin\left(\frac{3}{32}n\pi\right)}{\left(\frac{3}{32}n\pi\right)}\right)\left[1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{2}} + e^{-j\frac{3}{4}n\pi}\right]$$

Thus, the spectrum for Unipolar RZ with four alternate "1" and "0"'s is

$$|W(f)| = \sum_{n=-\infty}^{\infty} \frac{3}{32} A \left|\frac{\sin\left(\frac{3}{32}n\pi\right)}{\left(\frac{3}{32}n\pi\right)}\right| \left|1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{2}} + e^{-j\frac{3}{4}n\pi}\right| \delta\left(f - \frac{n}{8}R\right)$$

3-18

(a) Substituting (3-40) into (3-36a) the PSD for Polar RZ signaling is



$$\mathcal{P}(f) = \frac{A^2}{T_b} |F(f)|^2$$

where the pulse shape, f(t), is shown in the figure. Thus,

$$F(f) = \mathcal{F}[f(t)] = \tau \frac{\sin(\pi f \tau)}{(\pi f \tau)}$$

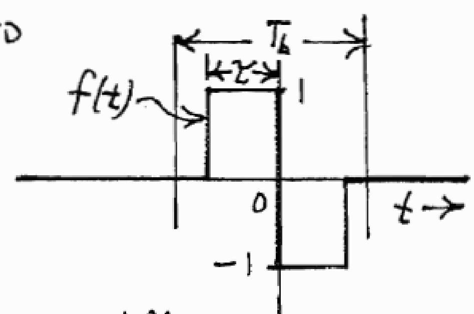
and $$\mathcal{P}(f) = \frac{A^2 \tau^2}{T_b} \left[\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right]^2$$

For the case of $\tau = \frac{1}{2} T_b$, this becomes

$$\mathcal{P}(f) = \frac{A^2 T_b}{4} \left[\frac{\sin(\frac{\pi}{2} f T_b)}{(\frac{\pi}{2} f T_b)}\right]^2$$

The first-null bandwidth is $B_{null} = \frac{2}{T_b} = 2R$ and the bandwidth efficiency is $\eta = \frac{1}{2}$ (bit/sec)/Hz.

(b) Equation (3-36) can also be used to evaluate the PSD for RZ Manchester signaling where the pulse shape is shown in the figure.



$$F(f) = \tau \left(\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right) \left[e^{j\omega \frac{\tau}{2}} - e^{-j\omega\frac{\tau}{2}}\right]$$

3-18 (Continued)

$$\Rightarrow \quad F(f) = j2\,\tau\left(\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right)\sin\left(\frac{\omega\tau}{2}\right)$$

Using (3-40) and (3-36), the PSD for Manchester signaling is

$$\mathcal{P}(f) = \frac{4A^2\tau^2}{T_b}\left[\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right]^2\left[\sin(\pi f \tau)\right]^2$$

If $\tau = \frac{1}{4}T_b$, this becomes

$$\underline{\underline{\mathcal{P}(f) = \frac{1}{4}A^2 T_b\left[\frac{\sin\left(\frac{\pi}{4} f T_b\right)}{\left(\frac{\pi}{4} f T_b\right)}\right]^2\left[\sin\left(\frac{\pi}{4} f T_b\right)\right]^2}}$$

The first-null bandwidth is $B_{null} = \frac{4}{T_b} = 4R$ and the spectral efficiency is $\eta = \frac{1}{4}$ (bits/sec)/Hz.

3-20

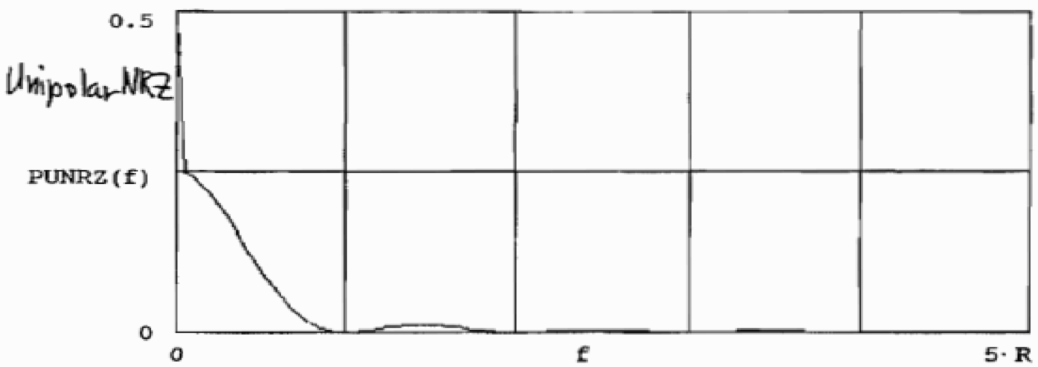
$$A := 1 \qquad R := 1 \qquad f := 0, 0.05 .. 5 \qquad Tb := \frac{1}{R}$$

$$Sa(x) := \text{if}\left[x \neq 0, \frac{\sin(x)}{x}, 1\right]$$

The PSD for Unipolar NRZ is given by (3-39b) and consists of both a continuous spectrum and a discrete spectrum. The computer cannot plot infinite values for the delta functions, so plot the weights of the delta functions instead. Thus (3-39b) will be broken into two functions, one for the continuous spectral plot and one for the discrete spectral plot.

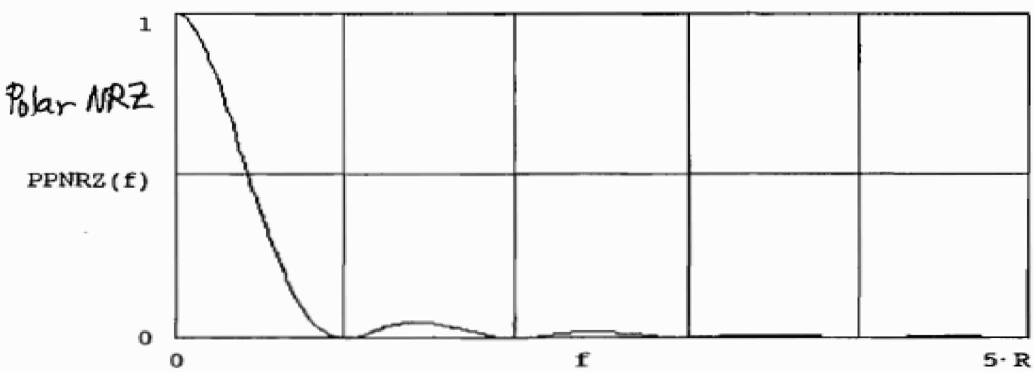
$$PUNRZc(f) := \left[A^2 \cdot \frac{Tb}{4}\right] \cdot (Sa(\pi\ f\ Tb))^2 \qquad PUNRZd(f) := \text{if}\left[f \neq 0, 0, \frac{A^2}{4}\right]$$

$$PUNRZ(f) := PUNRZc(f) + PUNRZd(f)$$



Use (3-41) for Polar NRZ spectrum:

$$PPNRZ(f) := \left[A^2\ Tb\right]\ (Sa(\pi\ f \cdot Tb))^2$$



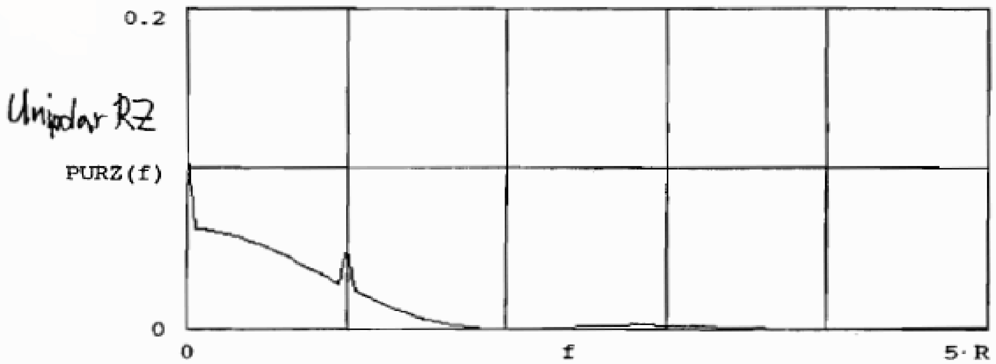
The PSD for Unipolar RZ is given by (3-43) and consists of both a continuous spectrum and a discrete spectrum. The computer cannot plot infinite values for the delta functions, so plot the weights of the delta functions instead. Thus (3-43) will be broken into two functions, one for the continuous spectral plot and one for the discrete spectral plot.

$$PURZc(f) := \left[A^2\ \frac{Tb}{16}\right] \left[Sa\left[\pi\ f\ \frac{Tb}{2}\right]\right]^2$$

$$PURZd(f) := \text{if}\left[\text{mod}(f,R) \neq 0, 0, \frac{A^2}{16} \cdot \left[Sa\left[\pi\ f\ \frac{Tb}{2}\right]\right]^2\right]$$

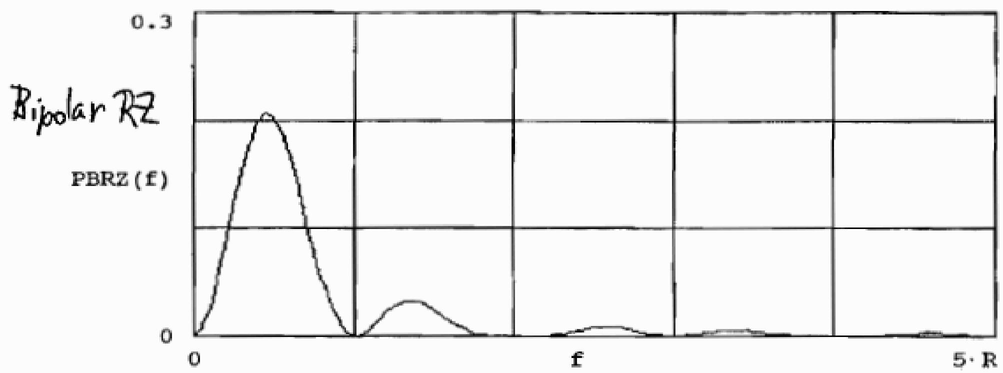
3-20 (Continued)

$$PURZ(f) := PURZc(f) + PURZd(f)$$



Using (3-45) the PSD for Bipolar RZ is:

$$PBRZ(f) := A^2 \left[\frac{Tb}{4}\right]\cdot\left[Sa\left[\pi\ f\cdot\frac{Tb}{2}\right]\right]^2 \ (\sin(\pi\ f\ Tb))^2$$

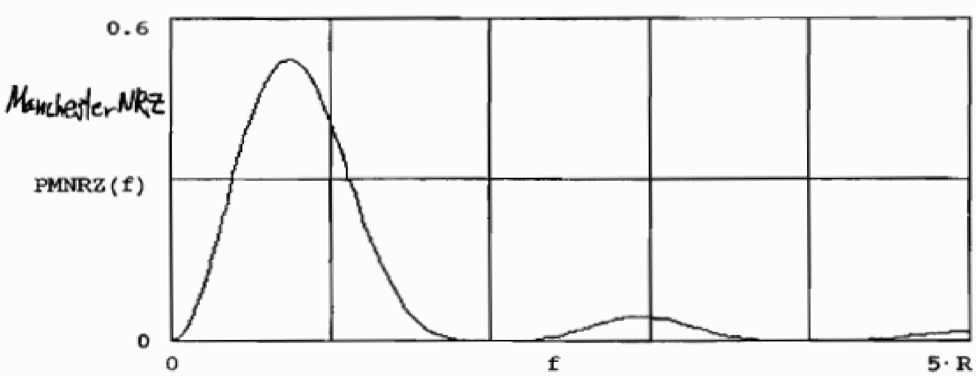


$A := 1$ $\quad R := 1$ $\quad f := 0, 0.05 .. 5$ $\quad Tb := \frac{1}{R}$

$$Sa(x) := \text{if}\left[x \neq 0, \frac{\sin(x)}{x}, 1\right]$$

Use (3-46c) for the Manchester NRZ spectrum:

$$PMNRZ(f) := A^2\cdot Tb\left[Sa\left[\pi\cdot f\cdot\frac{Tb}{2}\right]^2\right]\cdot\left[\sin\left[\pi\ f\cdot\frac{Tb}{2}\right]^2\right]$$



3-27

$$L = 8 = 2^{\ell} \Rightarrow \ell = 3$$

(a) $D = \frac{R}{\ell} = \frac{9600 \text{ bits/sec}}{3 \text{ bits/symbol}} = \underline{\underline{3.2k \text{ symbol/sec}}}$

(b) $D = \frac{2B}{1+r} = \frac{2(2.4k)}{1+r} = 3.2k \Rightarrow \underline{\underline{r = 0.5}}$

3-35

$$M = 16 = 2^4 \Rightarrow n = 4$$

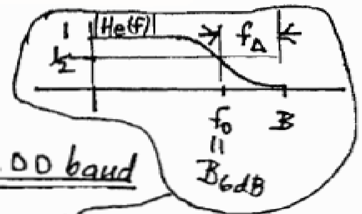
(a) Binary PCM $\Rightarrow \ell = 1$, $R = nf_s = 4f_s = D$

$$D = \frac{2B}{1+r} = \frac{2(4kHz)}{1+0.5} = \underline{\underline{5.33 \text{ kbits/sec}}}$$

(b) From (a) $f_s = \frac{D}{4} = \frac{5.33k}{4} = 1.33 kHz$

$$B_{\substack{analog \\ max}} = \frac{f_s}{2} = \frac{1.33k}{2} = \underline{\underline{667 \text{ Hz}}}$$

3-36



(a) $L = 2^\ell = 4 \Rightarrow \ell = 2$

$$D = R/\ell = \frac{2400}{2} = \underline{\underline{1200 \text{ baud}}}$$

(b) $B = \frac{1}{2}(1+r)D$ where $r = \frac{f_\Delta}{f_0} = 0 \Rightarrow B = f_0 = B_{6dB}$

$$\Rightarrow B_{6dB} = \frac{1}{2}(1+0)D = \frac{1}{2}(1200) = \underline{\underline{600 \text{ Hz}}}$$

(c)

$$B_{absolute} = \frac{1}{2}(1+r)D = \frac{1}{2}(1+0.5)(1200) = \frac{3}{4}(1200)$$

$r = 0.5$

$$\Rightarrow \underline{\underline{B_{absolute} = 900 \text{ Hz}}}$$

3-41

(a.) From (3-84)

$$\delta = \frac{2\pi f_a A}{f_s} \quad ; \quad f_a = 3.4\text{ kHz} \;\; \& \;\; A = \frac{1}{2}$$

We need to determine the f_s which the channel can support. Assuming that a $r=0$ roll-off factor is used, then

$$f_s = D = 2B = 2(1\text{ MHz}) = 2 \times 10^6 \frac{\text{samples}}{\text{sec}}$$

$$\Rightarrow \delta = \frac{2\pi(3.4k)(\frac{1}{2})}{2 \times 10^6} = \underline{\underline{0.00534}}$$

(Note: The channel has to be equalized with a Nyquist filter.)

(b.)

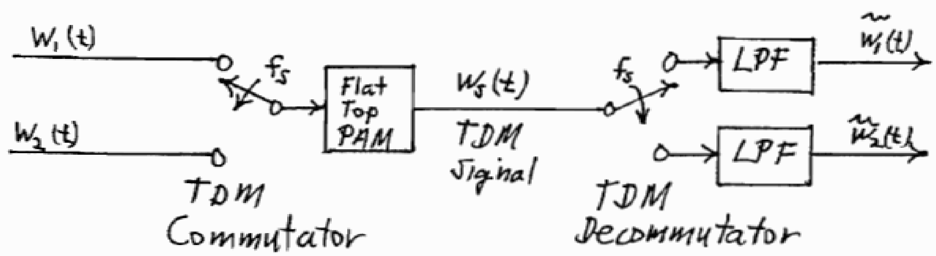
$$\delta = \frac{2\pi(3.4k)(\frac{1}{2})}{25 \times 10^3} = \underline{\underline{0.427}}$$

(Note: No Channel equalization required.)

3-44

(a.) Each analog signal has a highest frequency of $B = 3\,kHz$

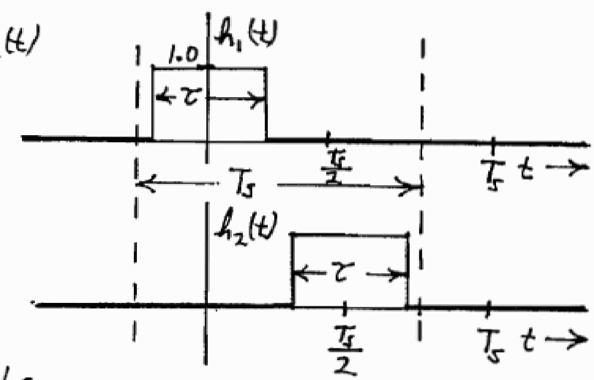
$\Rightarrow$ The minimum sampling frequency for each analog signal is $\underline{\underline{f_s = 2B = 6\,kHz}}$



(b.) Referring to (3-8), the sampled TDM signal is

$$w_s(t) = \sum_{k=-\infty}^{\infty} w_1(kT_s)\,h_1(t-kT_s) + \sum_{k=-\infty}^{\infty} w_2(kT_s)\,h_2(t-kT_s)$$

where $h_1(t)$ and $h_2(t)$ are shown in the figure and $\tau \le \frac{T_s}{2}$ and $f_s \ge 2B$.



Following the same procedure as described in (3-8) thru (3-13), the spectrum of the TDM instantaneously sampled (flat-topped) PAM signal is

$$W_s(f) = \frac{1}{T_s} H_1(f) \sum_{k=-\infty}^{\infty} W_1(f-kf_s) + \frac{1}{T_s} H_2(f) \sum_{k=-\infty}^{\infty} W_2(f-kf_s)$$

where $H_1(f) = \tau\left(\frac{\sin(\pi f\tau)}{\pi f\tau}\right)$ and $H_2(f) = \tau\left(\frac{\sin(\pi f\tau)}{\pi f\tau}\right) e^{-j2\pi f\frac{T_s}{2}}$

$$\Rightarrow W_s(f) = f_s\,\tau\,\frac{\sin(\pi f\tau)}{\pi f\tau} \sum_{k=-\infty}^{\infty} \Pi\left(\frac{f-kf_s}{2B}\right) + 2B\tau\left(\frac{\sin(\pi f\tau)}{(\pi f\tau)}\right)$$

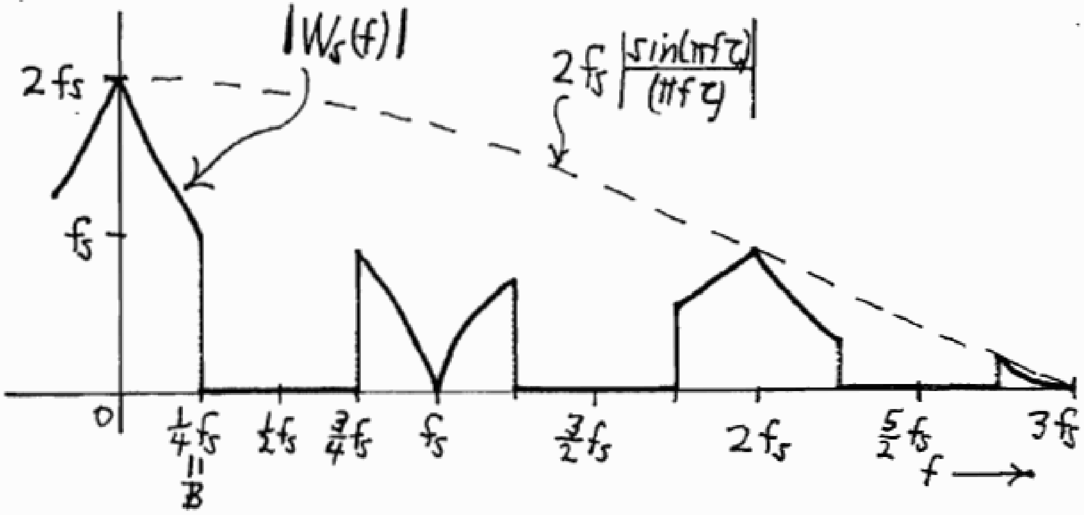
$$+ 2B\tau\left(\frac{\sin(\pi f\tau)}{(\pi f\tau)}\right) e^{-j\pi T_s f} \sum_{k=-\infty}^{\infty} \Lambda\left(\frac{f-kf_s}{B}\right)$$

3-44

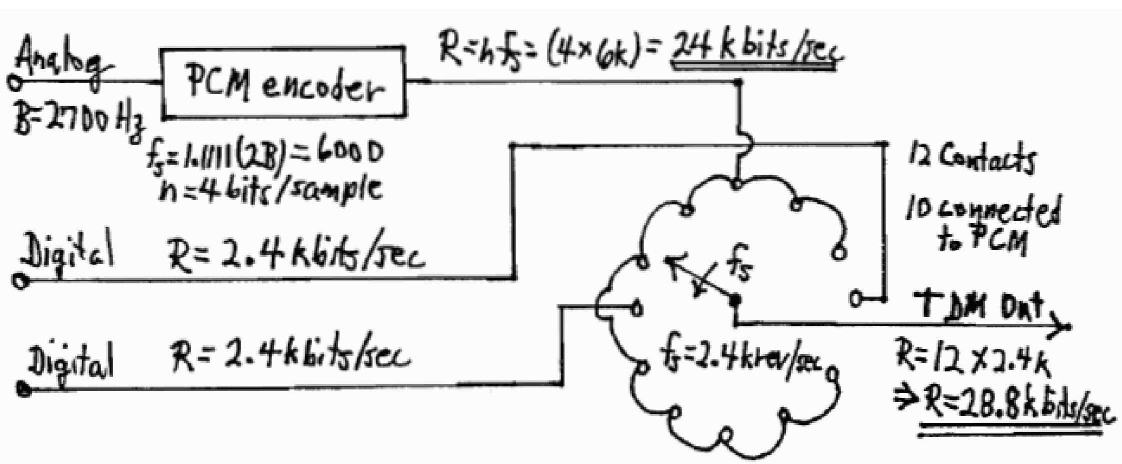
Thus,

$$\underline{\underline{|W_s(f)| = f_s \left|\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right| \sum_{k=-\infty}^{\infty} \left| \Pi\left(\frac{f-kf_s}{2B}\right) + e^{j\pi T_s f} \Lambda\left(\frac{f-kf_s}{B}\right)\right|}}$$

For the sketch, let the parameters be the same as those shown in Fig. 3-6. Let $\tau/T_s = 1/3$, $f_s = 4B$. Using a programable calculator, the following sketch is obtained.



3-46



Chapter 4

4-2

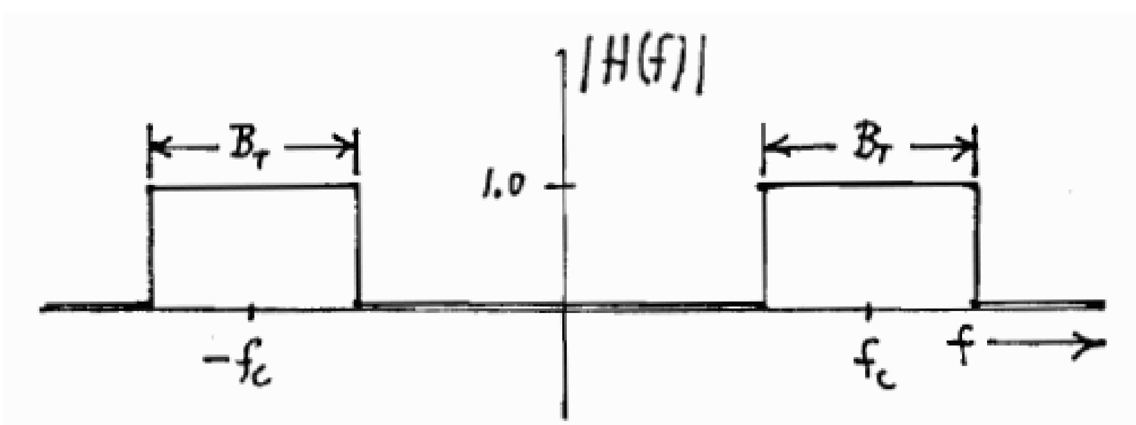
Using (2-26) with the help of Sec. A-5,

$$G(f) = A_c M(f) = 50\left[-j\delta(f-1000) + j\delta(f+1000)\right]$$

Substituting this into (4-15) and using $\delta(-f) = \delta(f)$, the voltage spectrum of this DSB-SC signal is

$$S(f) = -j25\,\delta(f-f_c-1000) + j25\,\delta(f-f_c+1000) - j25\,\delta(f+f_c-1000) + j25\,\delta(f+f_c+1000)$$

4-3



4-3 (Continued)

(b) $v_2(t) = \mathrm{Re}\{g_2(t)e^{j\omega_c t}\}$

where $g_2(t) = \frac{1}{2}g_1(t) * k(t) \leftrightarrow G_2(f) = \frac{1}{2}G_1(f)K(f)$

and $k(t) = \mathcal{F}^{-1}[K(f)]$

Also, $H(f) = \frac{1}{2}\left[K(f-f_c) + K^*(-f-f_c)\right]$

$$\Rightarrow \quad K(f) = \left\{\begin{array}{ll} 2, & |f| < B_T/2 \\ 0 & , f \text{ elsewhere} \end{array}\right\}$$

Evaluate $k(t)$:

$$k(t) = \int_{-\frac{B_T}{2}}^{\frac{B_T}{2}} 2e^{j2\pi ft}\,df = 2B_T \frac{\sin(\pi B_T t)}{(\pi B_T t)}$$

Know that $g_1(t) = A\,\Pi\!\left(\frac{t}{T}\right) = A\left\{\begin{array}{l} 1, |t| < T/2 \\ 0, t \text{ elsewhere} \end{array}\right\}$

$$\Rightarrow g_2(t) = \frac{1}{2}g_1(t) * k(t) = \frac{1}{\cancel{2}}\int_{-\frac{T}{2}}^{\frac{T}{2}} A\cancel{2}B_T \frac{\sin[\pi B_T(t-\lambda)]}{[\pi B_T(t-\lambda)]}\,d\lambda$$

Let $\lambda_1 = \pi B_T(t-\lambda) \Rightarrow d\lambda_1 = -\pi B_T\,d\lambda$

$$\Rightarrow g_2(t) = A\cancel{B_T}\int_{\pi B_T(t+\frac{T}{2})}^{\pi B_T(t-\frac{T}{2})} \frac{\sin\lambda_1}{\lambda_1}\left(-\frac{1}{\pi \cancel{B_T}}\,d\lambda_1\right)$$

$$= \frac{A}{\pi}\left[-\int_{\pi B_T(t+\frac{T}{2})}^{0} \frac{\sin\lambda_1}{\lambda_1}\,d\lambda_1 - \int_{0}^{\pi B_T(t-\frac{T}{2})} \frac{\sin\lambda_1}{\lambda_1}\,d\lambda_1\right]$$

$$\Rightarrow \quad g_2(t) = \frac{A}{\pi}\left\{+\,Si\left[\pi B_T\left(t+\frac{T}{2}\right)\right] - Si\left[\pi B_T\left(t-\frac{T}{2}\right)\right]\right\}$$

and $v_2(t) = \mathrm{Re}\{g_2(t)e^{j\omega_c t}\}$

4-3 (Continued)

Thus,
$$v_2(t) = \frac{A}{\pi}\left\{ Si\left[\pi B_T\left(t+\frac{T}{2}\right)\right] - Si\left[\pi B_T\left(t-\frac{T}{2}\right)\right]\right\}\cos(\omega_c t)$$

(c.) When $B_T = \frac{4}{T}$

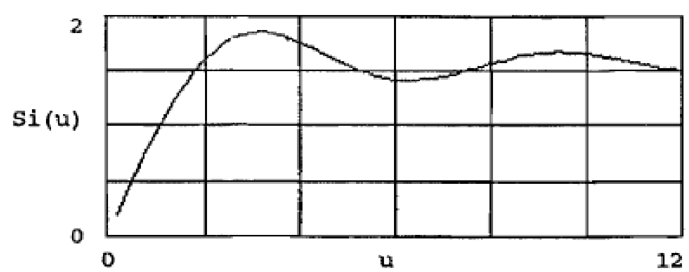
$$v_2(t) = \frac{A}{\pi}\left\{ Si\left[2\pi\left(\frac{2t}{T}+1\right)\right] - Si\left[2\pi\left(\frac{2t}{T}-1\right)\right]\right\}\cos(\omega_c t)$$

This is plotted with the help of the Si(u) function. (See p. 232 of Abramowitz and Stegun for a description of the Si(u) function.)

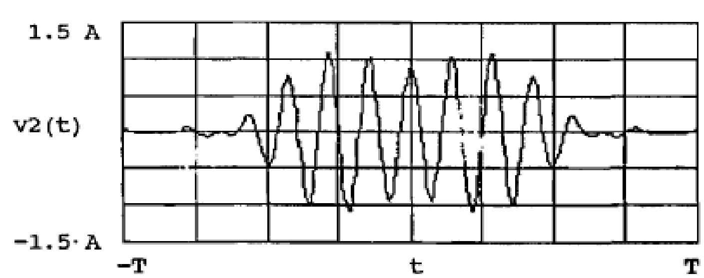
Using MathCAD we get:

$A := 1 \qquad T := 1 \qquad \omega := 2\,\pi\cdot 7 \qquad u := 0, 0.2\,..12$

$$Si(u) := \int_{0.001}^{u} \frac{\sin(x)}{x}\,dx \qquad t := -T, -T + 0.01\,..T$$



$$v2(t) := \left[\frac{A}{\pi}\right]\cdot\left[Si\left[2\,\pi\left[2\cdot\frac{t}{T}+1\right]\right] - Si\left[2\cdot\pi\cdot\left[2\cdot\frac{t}{T}-1\right]\right]\right]\cos(\omega\, t)$$



4-6

(a.) $s(t) = \mathrm{Re}\{500\, e^{j\omega_c t}\} + \mathrm{Re}\{-j100\, e^{j(\omega_c+\omega_a)t} + j100\, e^{j(\omega_c-\omega_a)t}\}$

$$s(t) = \mathrm{Re}\left\{500\left[1 - j(2j)\frac{100}{500}\left(\frac{e^{j\omega_a t} - e^{-j\omega_a t}}{2j}\right)\right] e^{j\omega_c t}\right\}$$

$$= \mathrm{Re}\left\{500\,[\underbrace{1 + \tfrac{2}{5}\sin(\omega_a t)}_{1+m(t)}]\, e^{j\omega_c t}\right\} \qquad AM$$

$$\Rightarrow \underline{\underline{g(t) = 500 + 200\sin(\omega_a t)}}\ , \quad \underline{\underline{m(t) = 0.4\sin(\omega_a t)}}$$

(b.) $x(t) = \mathrm{Re}\{g(t)\} = \underline{\underline{500 + 200\sin(\omega_a t)}}$, $y(t) = \mathrm{Im}\{g(t)\} = \underline{\underline{0}}$

(c.) $R(t) = |g(t)| = \underline{\underline{500 + 200\sin(\omega_a t)}}$, $\theta(t) = \angle g(t) = \underline{\underline{0^\circ}}$

(d.) $$P = \frac{\frac{1}{2}\langle |g(t)|^2\rangle}{50} = \frac{1}{100}\left[(500)^2 + 2\times10^5 \cancelto{0}{\langle \sin\omega_a t\rangle} + (200)^2 \cancelto{\frac{1}{2}}{\langle \sin^2(\omega_a t)\rangle}\right]$$

$$\Rightarrow \underline{\underline{P = 2{,}700 \text{ watts}}}$$

4-9

$$B := 100 \qquad f := 10, 20 \,..\, 1000 \qquad rad \equiv 1 \qquad deg \equiv \frac{rad}{\pi}\, 180$$

$$H1(f) := \frac{1}{\left[1 - \left[\frac{f}{B}\right]^2\right] + j\,\left[\sqrt{2}\,\frac{f}{B}\right]}$$

$$\theta 1(f) := \arg(H1(f))$$

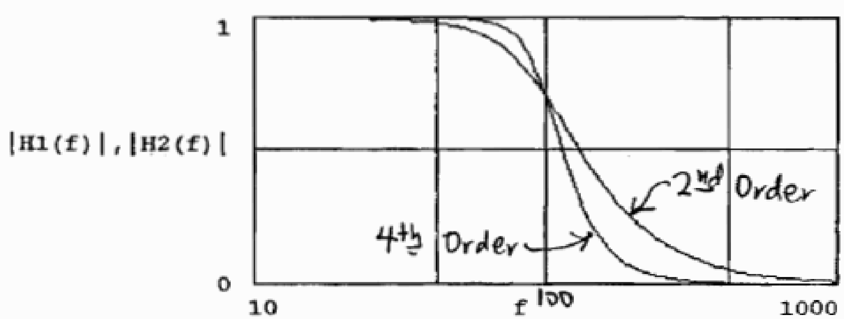
$$Td1(f) := \frac{1}{-2\cdot\pi\cdot f}\cdot\theta 1(f)$$

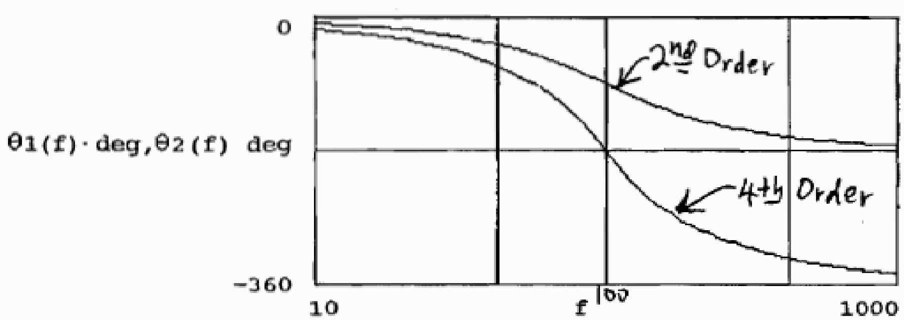
$$H2(f) := \frac{1}{\left[1 - \left[\frac{f}{B}\right]^2 + 0.765j\cdot\frac{f}{B}\right]}\cdot\frac{1}{\left[1 - \left[\frac{f}{B}\right]^2 + 1.848j\cdot\frac{f}{B}\right]}$$

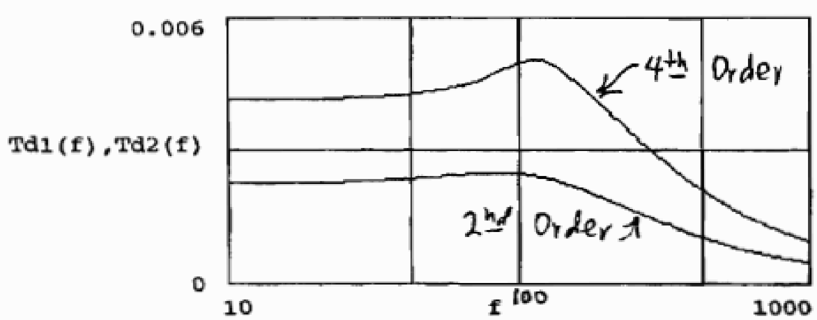
$$\theta(f) := \arg(H2(f))$$

$$\theta 2(f) := \mathrm{if}(f < 100, \theta(f), \theta(f) - 2\cdot\pi)$$

$$Td2(f) := \frac{1}{-2\cdot\pi\, f}\cdot\theta 2(f)$$







4-12

$t := 0, 0.1 .. 4 \qquad m := 1 .. 6$

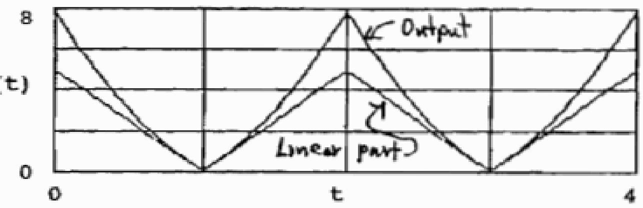
(a.)

$$vin(t) := \frac{1}{2} + \frac{4}{\pi^2} \cdot \left[\sum_m \left[\frac{\cos((2\cdot m - 1)\cdot \pi \cdot t)}{(2\,m-1)^2}\right]\right]$$

Note: vin(t) is the Fourier series for a triangle waveform.

$vout(t) := 5\cdot vin(t) + 1.5\cdot(vin(t))^2 + 1.5\cdot(vin(t))^3$

$vol(t) := 5\cdot vin(t)$ <-----Linear part of the output.



(b.)

$M := 5 \qquad N := 2^M \qquad N = 32 \qquad k := 0 .. N-1 \qquad T := 2$

$dt := \frac{T}{N} \qquad dt = 0.063 \qquad t_k := k\cdot dt$

$vo_k := vout(k\cdot dt) \qquad vol_k := vol(k\,dt) \qquad n := 0 .. N-1$

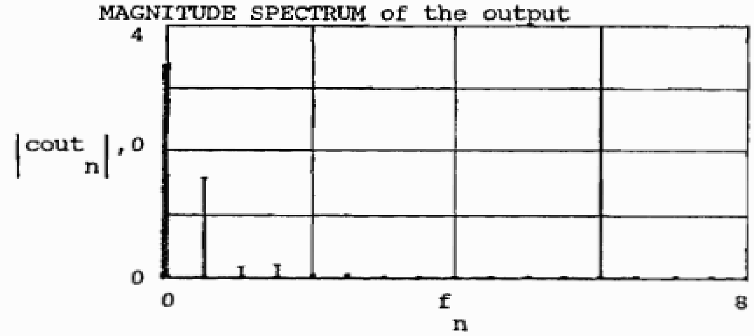
Since the signal is periodic, the spectrum will consist of delta functions (which can't be plotted directly since the delta function has an infinite value). However the weights of the delta functions are finite and can be plotted. The weights may be obtained from the complex Fourier series coefficients. Furthermore, the complex Fourier series coefficients may be calculated using the FFT by substituting (2-178) into (2-186). Thus,

$cout := \left[\frac{1}{\sqrt{N}}\right]\cdot icfft(vo) \qquad col := \left[\frac{1}{\sqrt{N}}\right] icfft(vol)$

$f_n := \frac{n}{T} \qquad f_1 = 0.5 \qquad fs := \frac{1}{dt} \qquad fs = 16$

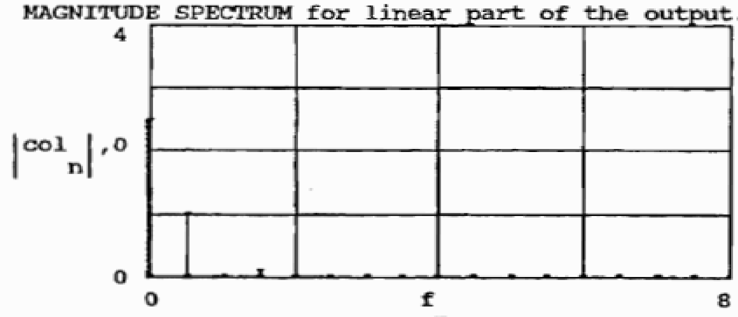
MAGNITUDE SPECTRUM of the output

$cout_0 = 3.375$



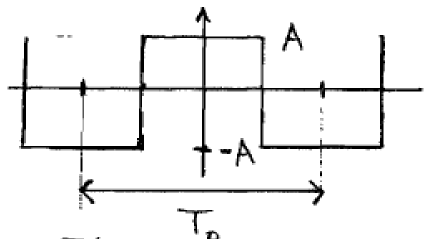
MAGNITUDE SPECTRUM for linear part of the output.

$col_0 = 2.5$



4-15

The output is a square wave as shown

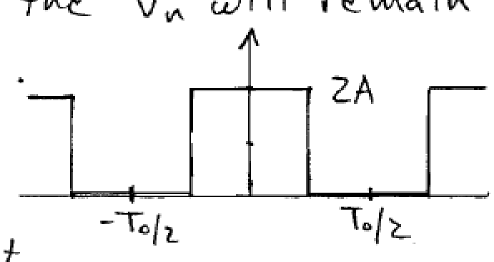


$$v(t) = \sum_{n=0}^{\infty} V_n \cos(n\omega_0 t)$$

where, using (2-96),

$$b_n = 0 \quad \text{and} \quad a_n = V_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} v(t)\cos(n\omega_0 t)\,dt$$

Since we are only interested in V_n for $n \geq 1$, we can shift the DC level of the wave-form to make the integral easier to evaluate and yet the V_n will remain the same for $n \neq 0$.



$$V_n = \frac{2}{T_0}\int_{-T_0/4}^{T_0/4} 2A\cos(n\omega_0 t)\,dt$$

$$= \frac{2}{T_0}(2A)\left.\frac{\sin(n\omega_0 t)}{n\omega_0}\right|_{-T_0/4}^{T_0/4} = \frac{4A}{n2\pi}\,2\sin\left(\frac{n\pi}{2}\right)$$

$$= \frac{4A}{n\pi}\begin{cases} 0, & n = \text{even} \\ +1, & n = 1, 5, 9, \ldots \\ -1, & n = 3, 7, 11, \ldots \end{cases}$$

$$\Rightarrow V_n^2 = \begin{cases} 0, & n \text{ even} \\ \left(\frac{4A}{n\pi}\right)^2, & n \text{ odd} \end{cases}$$

Using (4-47)

$$THD\,\% = \sqrt{\frac{\sum_{n=2}^{\infty} V_n^2}{V_1^2}} \times 100 = \sqrt{\frac{\sum_{n=3}^{\infty}\left(\frac{4A}{n\pi}\right)^2}{\left(4A/\pi\right)^2}}$$

4-15 (Continued)

$$THD\% = \sqrt{\sum_{\substack{n=3 \\ n=\text{odd}}}^{\infty} \frac{1}{n^2}} \times 100 = \underline{\underline{48.3\%}}$$

Using programable calculator

Check:

$$THD\% = \sqrt{\frac{\text{Total Power} - \left(\frac{V_1}{\sqrt{2}}\right)^2}{\left(\frac{V_1}{\sqrt{2}}\right)^2}} \times 100$$

$$= \sqrt{\frac{A^2 - \left(\frac{4A}{\pi\sqrt{2}}\right)^2}{\left(\frac{4A}{\pi\sqrt{2}}\right)^2}} \times 100$$

$$= \sqrt{\frac{\pi^2 - 8}{8}}\,(100) = \sqrt{0.2337}\,(100) = \underline{\underline{48.3\%}}$$

4-18

$$s(t) = A_c[m(t)\cos\omega_c t \mp \hat{m}(t)\sin\omega_c t]$$
$$= \text{Re}\{A_c[m(t) \pm j\hat{m}(t)]\, e^{j\omega_c t}\}$$

$$\Rightarrow g(t) = A_c[m(t) \pm j\,\hat{m}(t)] = R(t)\,\underline{/\theta(t)}$$

Output of Envelope Detector is

$$v_{out}(t) = KR(t) = K|g(t)|$$

$$\Rightarrow v_{out}(t) = KA_c\sqrt{m^2(t) + \hat{m}^2(t)} \neq K\,m(t)$$

The output is $\underline{\underline{\text{distorted}}}$.

4-21

$$\frac{d\theta_e(t)}{dt} = \frac{d\theta_i(t)}{dt} - K_d k_v \theta_e(t) * f(t)$$

$$\Rightarrow \quad s\,\Theta_e(s) = s\,\Theta_i(s) - k_d k_v \Theta_e(s) F(s)$$

or

$$\Theta_e(s) = \frac{s\,\Theta_i(s)}{s + k_d k_v F(s)}$$

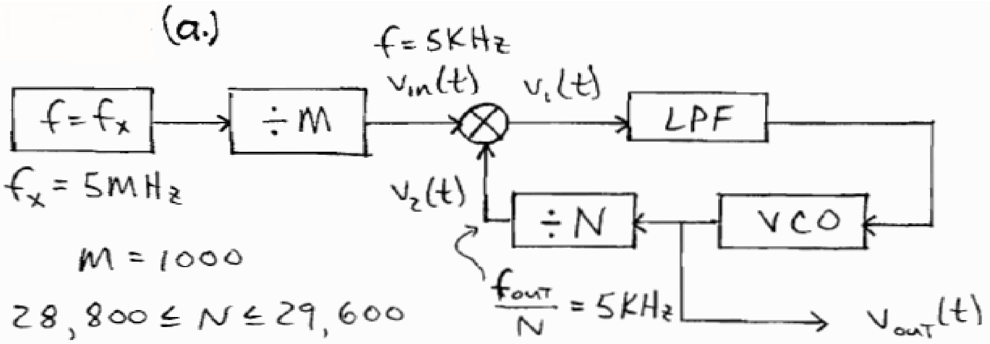
Final value theorem:

$$\lim_{t\to\infty} \theta_e(t) = \lim_{s\to 0}\left[s\Theta_e(s)\right] = \lim_{s\to 0} \frac{s^2\,\Theta_i(s)}{s + k_d k_v F(s)}$$

$$\Rightarrow \text{If } F(0) \neq 0 \Rightarrow \underline{\underline{\lim_{t\to\infty} \theta_e(t) = 0}}$$

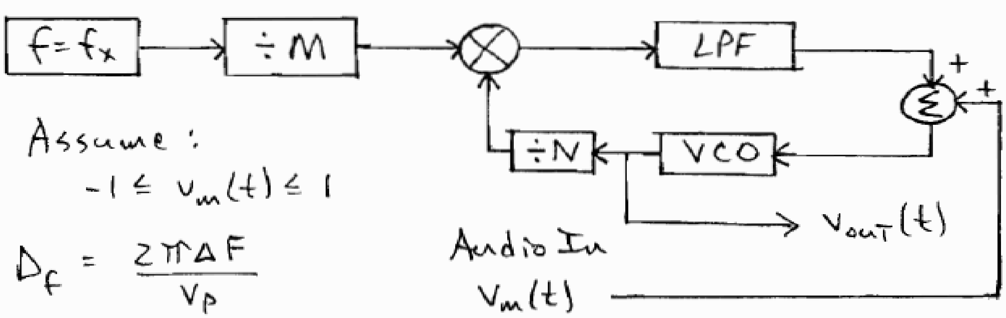
4-23

(a.)



(b.)

$144 \text{ MHz} \le f_{out} \le 148 \text{ MHz}$



Assume:

$-1 \le v_m(t) \le 1$

$$D_f = \frac{2\pi \Delta F}{V_p}$$

$$= 2\pi(5000)\ \frac{\text{rad.}}{\text{volt}\cdot\text{sec}}$$

4-26

(a.) $f_{LO} = 96.9 + 10.7 = \underline{107.6\,MHz}$

(b.) RF: Flat bandpass over 96.81 MHz to 96.99 MHz and reject image frequency of 118.3 MHz

IF: Flat bandpass over 10.61 MHz to 10.79 MHz and reject adjacent channel signals on each side of this bandpass

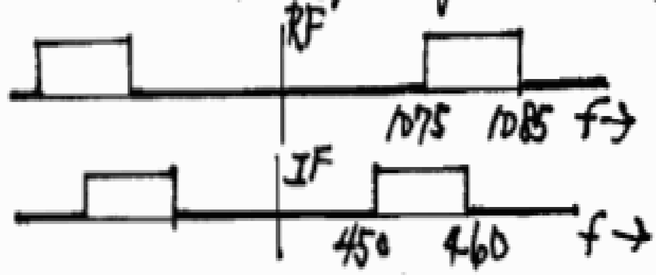
(c.) $f_{image} = f_c + 2f_{if} = 96.9 + 2(10.7) = \underline{\underline{118.3\,MHz}}$

4-30

(a.) RF filter $\ni f_c \pm B/2$; IF filter $\ni f_{if} \pm B/2$ where $B = 10\,kHz$ be since the AM channel spacing is 10 kHz.

RF: $1080 \pm 5\,kHz$

IF: $455 \pm 5\,kHz$



(b.) $f_{image} = f_c + 2f_{if} = 1990$ kHz