Chapter 6

6-2

(a.)

$$\overline{x(t)} = \int_0^{\pi/2} \frac{2}{\pi} \cdot A \cos(\omega_0 t + \Theta)\, d\Theta$$

$$= \frac{2A}{\pi} \sin(\omega_0 t + \Theta) \Big|_0^{\pi/2}$$

$$= \frac{2A}{\pi} \left[\sin(\omega_0 t + \pi/2) - \sin \omega_0 t \right]$$

$$= \frac{2A}{\pi} \left[\cos \omega_0 t - \sin \omega_0 t \right]$$

$$= \frac{2\sqrt{2}A}{\pi} \left[\cos\left(\frac{\pi}{4}\right) \cos \omega_0 t - \sin\left(\frac{\pi}{4}\right) \sin \omega_0 t \right]$$

$$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} = \sin \frac{\pi}{4}$$

$$= \frac{2\sqrt{2}A}{\pi} \cos\left(\omega_0 t + \frac{\pi}{4}\right)$$

(b.) $\overline{x(t)}$ is a function of time

$\therefore$ $x(t)$ is <u>not</u> stationary.

6-6

Ergodicity $\Rightarrow \langle [\ ] \rangle = \overline{[\ ]}$

(a.) $P = \overline{n^2(t)} = \overline{[n_1(t) + n_2(t)]^2}$

$= \overline{n_1^2(t)} + 2\,\overline{n_1(t)\,n_2(t)} + \overline{n_2^2(t)}$ $\quad 2\,\overline{n_1(t)\,n_2(t)} \to 0$ $\quad$ orthogonal

$= \overline{n_1^2(t)} + \overline{n_2^2(t)}$

$= 5 + 10 = \underline{\underline{15 \text{ Watts}}}$

(b.) $P = \overline{n^2(t)} = \overline{n_1^2(t)} + 2\,\overline{n_1(t)\,n_2(t)} + \overline{n_2^2(t)}$

uncorrelated $\quad 2\,\overline{n_1(t)\,n_2(t)} \to 2\,\overline{n_1(t)}\;\overline{n_2(t)}$

$P = 5 + 2(-2)(1) + 10 = \underline{\underline{11 \text{ Watts}}}$

(c.) $P = \overline{n^2(t)} = \overline{[n_1(t) + n_2(t)]^2}$

$= \overline{n_1^2(t)} + 2\,\overline{n_1(t)\,n_2(t)} + \overline{n_2^2(t)}$ $\quad 2\,\overline{n_1(t)\,n_2(t)} \to 2R_{n_1 n_2}(0)$

$= 5 + 2(2) + 10 = \underline{\underline{19 \text{ Watts}}}$

6-8

(a.) $\sin \omega_0 \tau$ ① $R(\tau) \neq R(-\tau)$ ×

$\underline{\underline{NO}}$

(b.) $\left(\dfrac{\sin \omega_0 \tau}{\omega_0 \tau}\right)$ ① $R(\tau) = R(-\tau)$

② $R(0) \geq |R(\tau)|$

$\underline{\underline{YES}}$ ③ $\mathcal{F}\left\{\dfrac{\sin \omega_0 \tau}{\omega_0 \tau}\right\}$ = Non-negative rectangle

(c.) $\cos \omega_0 \tau + \delta(\tau)$ ① $R(\tau) = R(-\tau)$

$\underline{\underline{YES}}$ ② $R(0) \geq |R(\tau)|$

③ $\mathcal{F}\left\{\cos \omega_0 \tau + \delta(\tau)\right\} = \frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0) + 1$

≥ 0

(d.) $e^{-a|\tau|}$ where $\underline{a < 0}$ ① $R(\tau) = R(-\tau)$

$\underline{\underline{NO}}$ ② $R(0) \ngeq |R(\tau)|$ ×

6-10

$$R_x(\tau) = 4e^{-\tau^2} + 3$$

(a.)

$$\mathcal{P}_x(f) = \mathcal{F}[R_x(\tau)] = \mathcal{F}[4e^{-\tau^2}] + \mathcal{F}[3] = \mathcal{F}\left[4e^{-\pi\left(\frac{\tau}{\sqrt{\pi}}\right)^2}\right] + \mathcal{F}[3]$$

$$\Rightarrow \mathcal{P}_x(f) \underset{\text{Using Table 2-2}}{=} 4\sqrt{\pi}\, e^{-\pi(f\sqrt{\pi})^2} + 3\delta(f) = \underline{\underline{4\sqrt{\pi}\, e^{-(\pi f)^2} + 3\delta(f)}}$$

(b.) This is a low-pass spectrum. $\Rightarrow$ Use (6-97) and (6-98).

$$\overline{f^2} = \frac{\int_{-\infty}^{\infty} f^2 \mathcal{P}_x(f)\,df}{\int_{-\infty}^{\infty} \mathcal{P}_x(\lambda)\,d\lambda} = \frac{4\int_{-\infty}^{\infty} f^2 \frac{1}{\sqrt{2\pi}\left(\frac{1}{\sqrt{2}\pi}\right)} e^{-f^2/2\left(\frac{1}{\sqrt{2}\pi}\right)^2} df + \overbrace{3\int_{-\infty}^{\infty} f^2\delta(f)\,df}^{0}}{4\left(\underbrace{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\left(\frac{1}{\sqrt{2}\pi}\right)} e^{-f^2/2\left(\frac{1}{\sqrt{2}\pi}\right)^2} df}_{1}\right) + 3\underbrace{\int_{-\infty}^{\infty}\delta(f)\,df}_{1}}$$

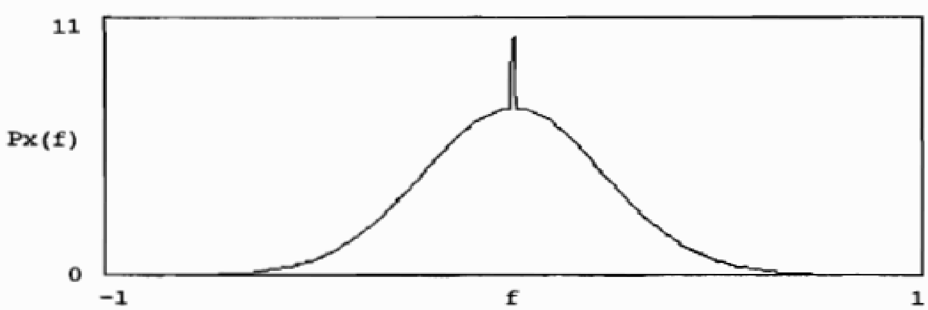
$$\Rightarrow \overline{f^2} = \frac{4\left(\frac{1}{\sqrt{2}\pi}\right)^2}{4+3} = \frac{\cancel{4}^{\,2}}{7}\,\frac{1}{\cancel{2}\pi^2} = \frac{2}{7\pi^2}$$

Thus, $B_{rms} = \sqrt{\overline{f^2}} = \sqrt{\frac{2}{7\pi^2}} = \sqrt{\frac{2}{7}}\,\frac{1}{\pi} = \underline{\underline{0.170\ Hz}}$

$\delta(0)$ cannot be plotted since it is infinity. Consequently, following the usual convention in EE, plot the WEIGHT of $\delta(f)$ instead at f=0.

f := -1,-0.99 ..1 $\qquad$ $\delta(f)$:= if(f $\approx$ 0,1,0)

$Px(f) := 4\sqrt{\pi}\cdot e^{-\left[(\pi\cdot f)^2\right]} + 3\cdot\delta(f)$ $\qquad$ Px(0) = 10.09



$Brms := \sqrt{\frac{2}{7}}\cdot\left[\frac{1}{\pi}\right]$ $\qquad$ Brms = 0.17

6-14

(a.) $x_{rms} = \sqrt{\overline{x^2(t)}}$

$$\overline{x^2(t)} = R_x(0) = \int_{-\infty}^{\infty} \mathcal{P}_x(f)\,df$$

$$= \int_{-B}^{0} \frac{1}{B}(B+f)\,df + \int_{0}^{B} \frac{1}{B}(B-f)\,df$$

$$= \frac{1}{B}\left[\left(Bf + \frac{f^2}{2}\right)\Big|_{-B}^{0} + \left(Bf - \frac{f^2}{2}\right)\Big|_{0}^{B}\right]$$

$$= \frac{1}{B}\left[-\left(-B^2 + \frac{B^2}{2}\right) + \left(B^2 - \frac{B^2}{2}\right)\right] = \frac{1}{B}\left[2B^2 - B^2\right]$$

$$= B \implies \underline{\underline{x_{rms} = \sqrt{B}}}$$

(b.) $\mathcal{P}_x(f) = \frac{1}{\sqrt{B}}\Pi\left(\frac{f}{B}\right) * \frac{1}{\sqrt{B}}\Pi\left(\frac{f}{B}\right)$

$$R_x(\tau) = \mathcal{F}^{-1}\left[\mathcal{P}_x(f)\right] = \frac{1}{B}\left\{\mathcal{F}^{-1}\left[\Pi\left(\frac{f}{B}\right)\right]\right\}^2$$

Table 2-1 · Multiplication property of $\mathcal{F}[\,]$

$$R_x(\tau) = \frac{1}{B}\left\{\mathcal{F}^{-1}\left[\Pi\left(\frac{f}{B}\right)\right]\right\}^2$$

Table 2-2

$$= \frac{1}{B}\left\{B\,\frac{\sin(\pi B\tau)}{\pi B\tau}\right\}^2$$

$$\underline{\underline{R_x(\tau) = B\left[\frac{\sin(\pi B\tau)}{\pi B\tau}\right]^2}}$$

6-16

(a.) $$\mathcal{P}_y(f) = |H(f)|^2 \mathcal{P}_n(f) = \left|\frac{K}{j2\pi f}\right|^2 \frac{N_0}{2}$$

$$\underline{\underline{\mathcal{P}_y(f) = \frac{N_0 K^2}{8\pi^2 f^2}}}$$

(b.) $$y_{rms}^2 = R_y(0) = \int_{-\infty}^{\infty} \mathcal{P}_y(f)\, df$$

$$= \int_{-\infty}^{\infty} \frac{N_0 K^2}{8\pi^2 f^2} = 2\int_0^{\infty} \left[\frac{N_0 K^2}{8\pi^2}\right] \frac{1}{f^2}\, df$$

$$= \frac{N_0 K^2}{4\pi^2}\left[\frac{-1}{f}\right]\Bigg|_0^{\infty} = \frac{N_0 K^2}{4\pi^2}\left[\frac{-1}{\infty} + \frac{1}{0}\right] = \underline{\underline{\infty}}$$

A practical integrator will have a large (i.e. finite) output.

6-18

From (6-95): $$\left(\frac{S}{N}\right)_{out} = \frac{2A_0^2 RC}{N_0[1 + (2\pi f_0 RC)^2]}$$

Let $RC = \tau$, $2\pi f_0 = \omega_0$

$$\Rightarrow \left(\frac{S}{N}\right)_{out} = \frac{2A_0^2 \tau}{N_0[1 + (\omega_0\tau)^2]}$$

For $\max\left[\left(\frac{S}{N}\right)_{out}\right]$, set $\dfrac{d\left[\left(\frac{S}{N}\right)_{out}\right]}{d\tau} = 0$

$$\Rightarrow \frac{d\left[\left(\frac{S}{N}\right)_{out}\right]}{d\tau} = \frac{[1 + (\omega_0\tau)^2]\,2A_0^2 - 2A_0^2\tau\,(2)\,(\omega_0\tau)\,\omega_0}{N_0\,[1 + (\omega_0\tau)^2]^2}$$

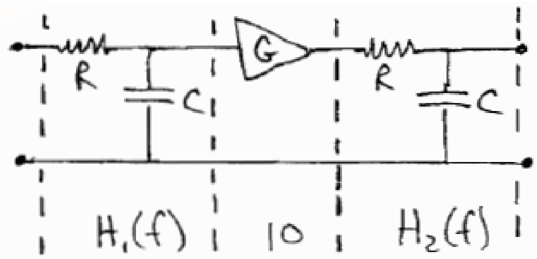
Set numerator $= 0$ $\Rightarrow 2A_0^2[1 + \omega_0^2\tau^2 - 2\tau\omega_0^2\tau] = 0$

$$\Rightarrow [1 - \omega_0^2\tau^2] = 0 \Rightarrow \omega_0^2\tau^2 = 1 \Rightarrow \tau^2 = \frac{1}{\omega_0^2} \Rightarrow \tau = \frac{1}{\omega_0}$$

Thus, $\underline{\underline{RC = \frac{1}{2\pi f_0}}}$ for $\max\left(\frac{S}{N}\right)_{out}$

6-22

(a.)



$$H_1(f) = H_2(f) = \frac{1}{1 + j\,f/f_0}$$

where $f_0 = \dfrac{1}{2\pi RC}$, and $G = 10$

$$H(f) = H_1(f)[10]\,H_2(f) = \underline{\underline{\frac{10}{[1 + j\,f/f_0]^2}}}$$

$$|H(f)| = \frac{10}{\left|1 + j\,2f/f_0 - (f/f_0)^2\right|} = \frac{10}{\sqrt{[1 - (f/f_0)^2]^2 + (2f/f_0)^2}}$$

Using a programmable calculator, find the value of $f = f_c$, such that:

$$|H(f_c)| = \frac{10}{\sqrt{2}} \implies \underline{\underline{f_c = 0.690\,f_0 \;;\; f_0 = \frac{1}{2\pi RC}}}$$

6-25

(a.) x_1 & y_2 uncorrelated $\xRightarrow{\text{prop. 3}}$ Independent

when $R_{xy}(\tau) = \overline{x(t_1)\,y(t_2)} = 10\sin(2\pi\tau) = \underline{0} = \overset{0}{\cancel{\bar{x}_1}}\,\overset{0}{\cancel{\bar{y}_2}}$

$\implies 2\pi\tau = \pm n\pi \Rightarrow$ <u>These r.v.'s are independent only when $t_2 - t_1 = \tau = \pm \dfrac{n}{2}$; $n = 0, 1, \ldots$</u>

6-25 (Continued)

$$10 \sin(2\pi \tau) = 10 \sin[2\pi(t_2 - t_1)] = \overline{x(t_1) y(t_2)}$$

This cannot be expressed as $\overline{x(t_1)}\ \overline{y(t_2)}$

$\therefore$ $x(t)$ and $y(t)$ are <u>not indep.</u>

6-27

(a.) Evaluate $\overline{x^2(t)} = R_x(0)$

$$\overline{x^2(t)} = \overline{A_o^2 \cos^2(\omega_o t + \theta)} = \frac{A_o^2}{2}\left[1 + \overline{\cos(2\omega_o t + 2\theta)}\right]$$

$$= \frac{A_o^2}{2} + \frac{A_o^2}{2} \int_0^{\pi/2} \cos(2\omega_o t + 2\theta) \frac{2}{\pi} d\theta$$

$$= \frac{A_o^2}{2} + \frac{A_o^2}{2} \left. \frac{\sin(2\omega_o t + 2\theta)}{2} \left(\frac{2}{\pi}\right) \right|_0^{\pi/2}$$

$$= \frac{A_o^2}{2} + \frac{A_o^2}{2\pi} \left[\sin(2\omega_o t + \pi) - \sin(2\omega_o t)\right]$$

$$= \frac{A_o^2}{2} + \frac{A_o^2}{2\pi} \left[-2 \sin(2\omega_o t)\right]$$

Using Sec. A-1

This is a function of t $\therefore$ $x(t)$ <u>not W.S.S.</u>

6-27 (Continued)

(b.)

Eqn. (6-42) $\mathcal{P}_x(f) = \lim_{T\to\infty}\left[\frac{\overline{|X_T(f)|^2}}{T}\right]$

where $X_T(f) = \int_{-T/2}^{T/2} x(t)\, e^{-j\omega t}\, dt$

$$= \int_{-T/2}^{T/2} A_o \cos(\omega_o t+\theta)\, e^{-j\omega t}\, dt$$

$$= A_o \int_{-T/2}^{T/2} \frac{e^{j(\omega_o t+\theta)} + e^{-j(\omega_o t+\theta)}}{2}\, e^{-j\omega t}\, dt$$

$$= \frac{A_o}{2} e^{j\theta} \int_{-T/2}^{T/2} e^{j(\omega_o-\omega)t}\, dt + \frac{A_o}{2} e^{-j\theta} \int_{-T/2}^{T/2} e^{-j(\omega_o+\omega)t}\, dt$$

$$= \frac{A_o}{2} e^{j\theta} \left.\frac{e^{j(\omega_o-\omega)t}}{j(\omega_o-\omega)}\right|_{-T/2}^{T/2} + \frac{A_o}{2} e^{-j\theta} \left.\frac{e^{-j(\omega_o+\omega)t}}{-j(\omega_o+\omega)}\right|_{-T/2}^{T/2}$$

$$= A_o\left[e^{j\theta}\, \frac{e^{j(\omega_o-\omega)T/2} - e^{-j(\omega_o-\omega)T/2}}{2j\,(\omega_o-\omega)} + e^{-j\theta}\, \frac{e^{j(\omega_o+\omega)T/2} - e^{-j(\omega_o+\omega)T/2}}{2j\,(\omega_o+\omega)}\right]$$

$$= A_o\, e^{j\theta}\, \frac{\sin(\omega_o-\omega)T/2}{(\omega_o-\omega)} + A_o\, e^{-j\theta}\, \frac{\sin(\omega_o+\omega)T/2}{(\omega_o+\omega)}$$

Let $x_1 = (\omega_o-\omega)T/2$ and $x_2 = (\omega_o+\omega)T/2$

$$= \frac{A_o T}{2}\left[e^{j\theta}\, \frac{\sin x_1}{x_1} + e^{-j\theta}\, \frac{\sin x_2}{x_2}\right]$$

$$\frac{|X_T(f)|^2}{T} = \frac{X_T(f)\, X_T^*(f)}{T} =$$

$$\frac{\left(\frac{A_o T}{2}\right)^2}{T}\left[e^{j\theta}\frac{\sin x_1}{x_1} + e^{-j\theta}\frac{\sin x_2}{x_2}\right]\left[e^{-j\theta}\frac{\sin x_1}{x_1} + e^{j\theta}\frac{\sin x_2}{x_2}\right]$$

6-27 (Continued)

$$\frac{\overline{|X_T(f)|^2}}{T} = \frac{A_o^2 T}{4}\left[\left(\frac{\sin x_1}{x_1}\right)^2 + \overline{e^{j2\theta}}\left(\frac{\sin x_1}{x_1}\right)\left(\frac{\sin x_2}{x_2}\right) + \overline{e^{-j2\theta}}\left(\frac{\sin x_1}{x_1}\right)\left(\frac{\sin x_2}{x_2}\right) + \left(\frac{\sin x_2}{x_2}\right)^2\right]$$

Aside:

$$\overline{e^{j2\theta}} = \int_0^{\pi/2} e^{j2\theta}\cdot\frac{2}{\pi}\,d\theta = j^2/\pi$$

$$\overline{e^{-j2\theta}} = \int_0^{\pi/2} e^{-j2\theta}\cdot\frac{2}{\pi}\,d\theta = -j^2/\pi$$

$$\frac{\overline{|X_T(f)|^2}}{T} = \frac{A_o^2}{4}\left[\frac{\overset{a}{T\pi}}{\pi}\left(\frac{\sin \pi T\overbrace{(f-f_o)}^{x}}{\pi T(f-f_o)}\right)^2 + \frac{T\pi}{\pi}\left(\frac{\sin \pi T(f+f_o)}{\pi T\underbrace{(f+f_o)}_{x'}}\right)^2\right]$$

From Sec. A-8

$$\delta(x) = \lim_{a\to\infty}\left[\frac{a}{\pi}\left(\frac{\sin ax}{ax}\right)^2\right]$$

$$\Rightarrow\quad \underline{\underline{\mathcal{P}_x(f) = \frac{A_o^2}{4}\left[\delta(f-f_o) + \delta(f+f_o)\right]}}$$

(c.) $\overline{x(t)} = A_o\,\overline{\cos(\omega_o t+\theta)} = A_o\cdot 0 = \underline{0}$

$$R_x(\tau) = \overline{x(t)\,x(t+\tau)}$$

$$= A_o^2\,\overline{\cos(\omega_o t+\theta)\cos(\omega_o t+\omega_o\tau+\theta)}$$

$$= \frac{A_o^2}{2}\cos\omega_o\tau + \frac{A_o^2}{2}\,\overline{\cos(2\omega_o t+\omega_o\tau+2\theta)} \to 0$$

$$= \frac{A_o^2}{2}\cos\omega_o\tau \quad ;\ \text{not a function of } t$$

$\therefore$ $\underline{\underline{x(t) \text{ is W.S.S.}}}$

6-30

$$s(t) = x(t)\cos(\omega_c t + \theta_c) - y(t)\sin(\omega_c t + \theta_c)$$

$$= s_{USSB}(t) + s_{LSSB}(t)$$

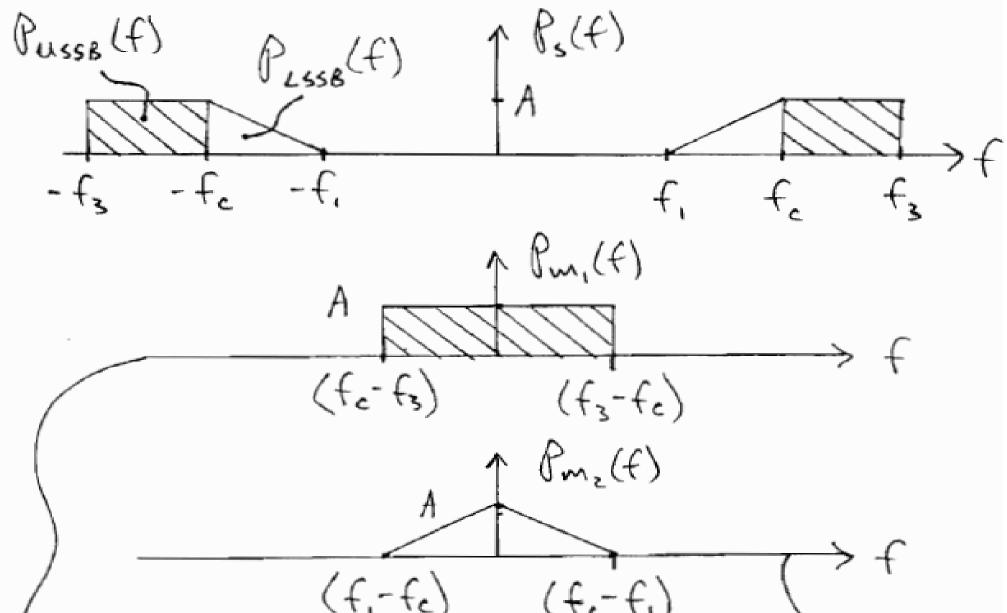
where

$$s_{USSB}(t) = m_1(t)\cos(\omega_c t + \theta_c) - \hat{m}_1(t)\sin(\omega_c t + \theta_c)$$

$$s_{LSSB}(t) = m_2(t)\cos(\omega_c t + \theta_c) + \hat{m}_2(t)\sin(\omega_c t + \theta_c)$$

$$\Rightarrow s(t) = [m_1(t) + m_2(t)]\cos(\omega_c t + \theta_c) - [\hat{m}_1(t) - \hat{m}_2(t)]\sin(\omega_c t + \theta_c)$$

$$\Rightarrow \underline{\underline{x(t) = m_1(t) + m_2(t)}}\ ; \quad \underline{\underline{y(t) = \hat{m}_1(t) - \hat{m}_2(t)}}$$



$$\underline{\underline{\mathcal{P}_{m_1}(f) = A\,\Pi\left(\frac{f}{2(f_3 - f_c)}\right)}}$$

$$\underline{\underline{\mathcal{P}_{m_2}(f) = A\,\Lambda\left(\frac{f}{f_c - f_1}\right)}}$$

6-32

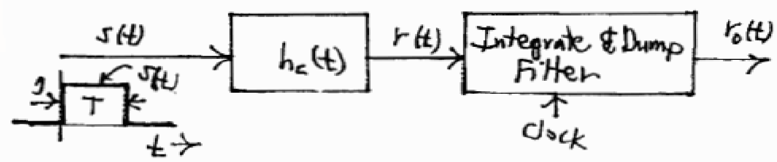
From example 6-9:

$$\mathcal{P}_v(f) = \frac{1}{4}\left[\mathcal{P}_x(f-f_c) + \mathcal{P}_x(-f-f_c)\right]$$

Where, from solution to problem 6-18.:

$$\mathcal{P}_x(f) = T_b\left[\frac{1-\cos(\pi f T_b)}{\pi f T_b}\right]^2 = \mathcal{P}_x(-f)$$

6-36



$$H_c(f) = \frac{B}{B+jf} = \frac{1}{1+j\left(\frac{f}{B}\right)} \qquad \text{Let } a = 2\pi B$$

Using Table 2-2

$$\Rightarrow h_c(t) = \begin{cases} 2\pi B e^{-2\pi B t}, & t>0 \\ 0, & t<0 \end{cases} = \begin{cases} a e^{-at}, & t>0 \\ 0, & t<0 \end{cases}$$

$$r(t) = s(t) * h_c(t) = \int_0^t s(\lambda) h_c(t-\lambda)\,d\lambda = \begin{cases} \int_0^t a e^{-a(t-\lambda)}\,d\lambda, & 0<t<T \\ \int_0^T a e^{-a(t-\lambda)}\,d\lambda, & t>T \end{cases}$$

$$\Rightarrow r(t) = \begin{cases} 1 - e^{-at}, & 0<t<T \\ e^{-at}\left[e^{aT} - 1\right], & t>T \end{cases}$$

$$r_o(t) = \begin{cases} \int_0^t r(\lambda)\,d\lambda, & 0<t<T \\ \int_T^t r(\lambda)\,d\lambda, & T<t<2T \end{cases} = \begin{cases} \int_0^t \left[1 - e^{-a\lambda}\right] d\lambda, & 0<t<T \\ (e^{aT}-1)\int_T^t e^{-a\lambda}\,d\lambda, & T<t<2T \end{cases}$$

$$\Rightarrow r_o(t) = \begin{cases} t + \frac{1}{a}\left(e^{-at} - 1\right), & 0<t<T \\ \frac{1}{a}\left(e^{aT}-1\right)\left(e^{-aT} - e^{-at}\right), & T<t<2T \end{cases}$$

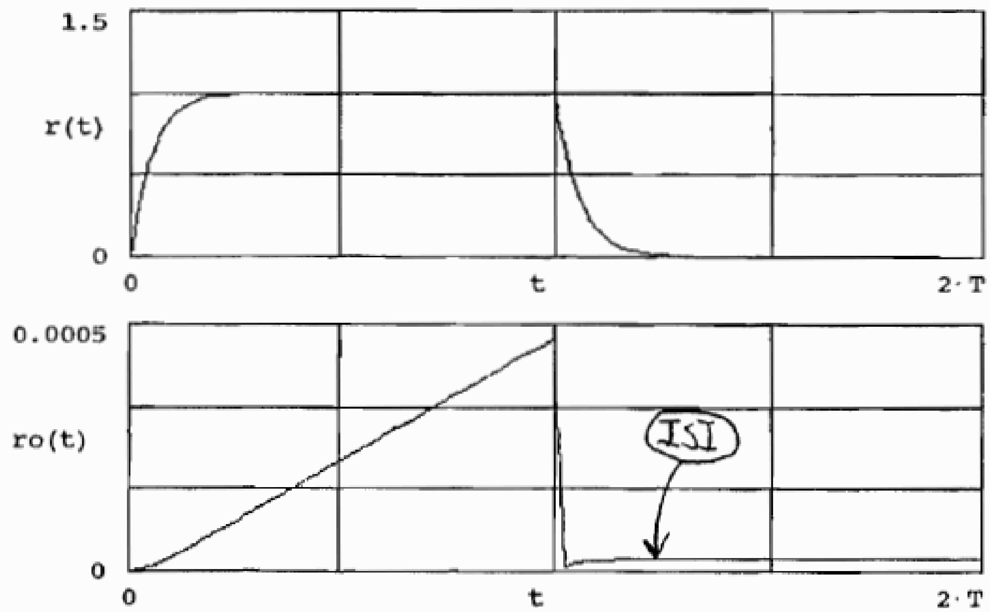
(a.)

$R := 2000 \qquad T := \frac{1}{R} \qquad B := 6000 \qquad a := 2\cdot\pi\cdot B \qquad t := 0, \frac{T}{40} \,..2\,T$

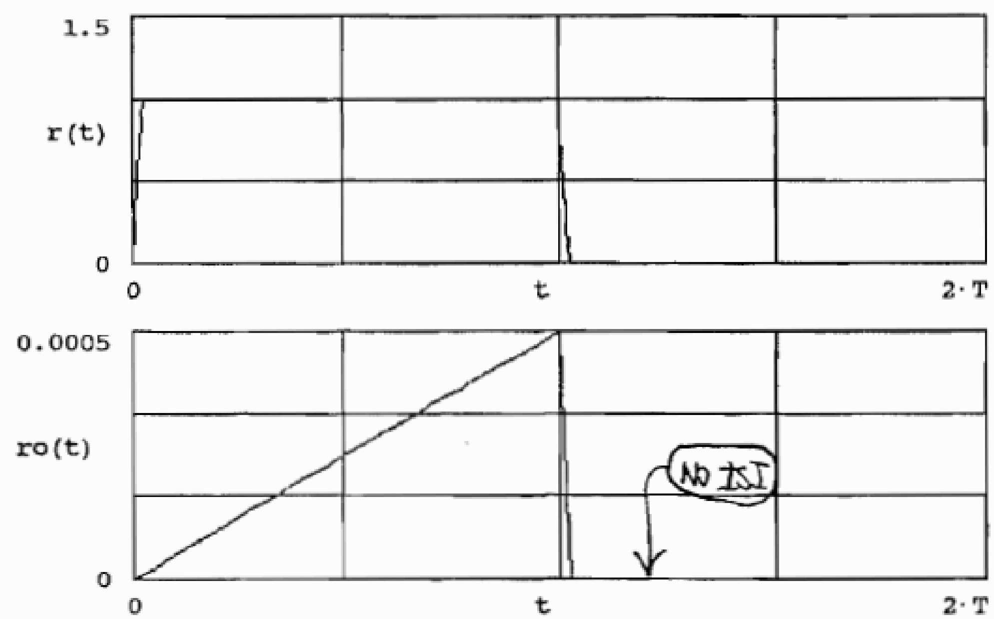
$r(t) := \text{if}\left[t \le T, 1 - e^{-a\,t}, \left[e^{a\,T} - 1\right]\cdot e^{-a\cdot t}\right]$

$ro(t) := \text{if}\left[t \le T, t + \frac{e^{-a\cdot t} - 1}{a}, \frac{e^{a\cdot T} - 1}{a}\cdot\left[e^{-a\,T} - e^{-a\cdot t}\right]\right]$

6-36 (Continued)



(b.) For all pass channel ∴ Let $B \to \infty$. ∴ Use $B = 100{,}000$.
Then, results are as follows.



Chapter 7

7-1

(a.) $r_o = \begin{cases} A + n_o, & s_1 \text{ sent} \\ -A + n_o, & s_2 \text{ sent} \end{cases}$

$$\Rightarrow f(r_o|s_1) = \frac{1}{\sqrt{2}\Delta_o} e^{-\frac{\sqrt{2}|r_o - A|}{\Delta_o}}$$

$$f(r_o|s_2) = \frac{1}{\sqrt{2}\Delta_o} e^{-\frac{\sqrt{2}|r_o + A|}{\Delta_o}}$$

Using (7-8)

$$P_e = \frac{1}{2}\int_{-\infty}^{V_T} \frac{1}{\sqrt{2}\Delta_o} e^{-\frac{\sqrt{2}|r_o - A|}{\Delta_o}} dr_o + \frac{1}{2}\int_{V_T}^{\infty} \frac{1}{\sqrt{2}\Delta_o} e^{-\frac{\sqrt{2}|r_o + A|}{\Delta_o}} dr_o$$

$m_{r_{o1}} = A$, $m_{r_{o2}} = -A$, the source probabilities are equally likely, and the conditional probabilities have symmetrical shapes about $\pm A$.

Thus $\boxed{V_T = 0}$.

$$\therefore P_e = \frac{1}{2\sqrt{2}\Delta_o}\left[\int_{-\infty}^{0} e^{-\frac{\sqrt{2}|r_o - A|}{\Delta_o}} dr_o + \int_{0}^{\infty} e^{-\frac{\sqrt{2}|r_o + A|}{\Delta_o}} dr_o\right]$$

$$= \frac{1}{2\sqrt{2}\Delta_o}\left[\int_{-\infty}^{0} e^{\frac{\sqrt{2}(r_o - A)}{\Delta_o}} dr_o + \int_{0}^{\infty} e^{-\frac{\sqrt{2}(r_o + A)}{\Delta_o}} dr_o\right]$$

Let $x_1 = \frac{\sqrt{2}(r_o - A)}{\Delta_o}$ and $x_2 = \frac{-\sqrt{2}(r_o + A)}{\Delta_o}$

$dx_1 = \frac{\sqrt{2}}{\Delta_o} dr_o$ $\qquad dx_2 = \frac{-\sqrt{2}}{\Delta_o} dr_o$

$$= \frac{1}{2\sqrt{2}\Delta_o}\left[\int_{-\infty}^{-\sqrt{2}A/\Delta_o} e^{x_1}\left(\frac{\Delta_o}{\sqrt{2}} dx_1\right) + \int_{-\sqrt{2}A/\Delta_o}^{-\infty} e^{x_2}\left(\frac{-\Delta_o}{\sqrt{2}} dx_2\right)\right]$$

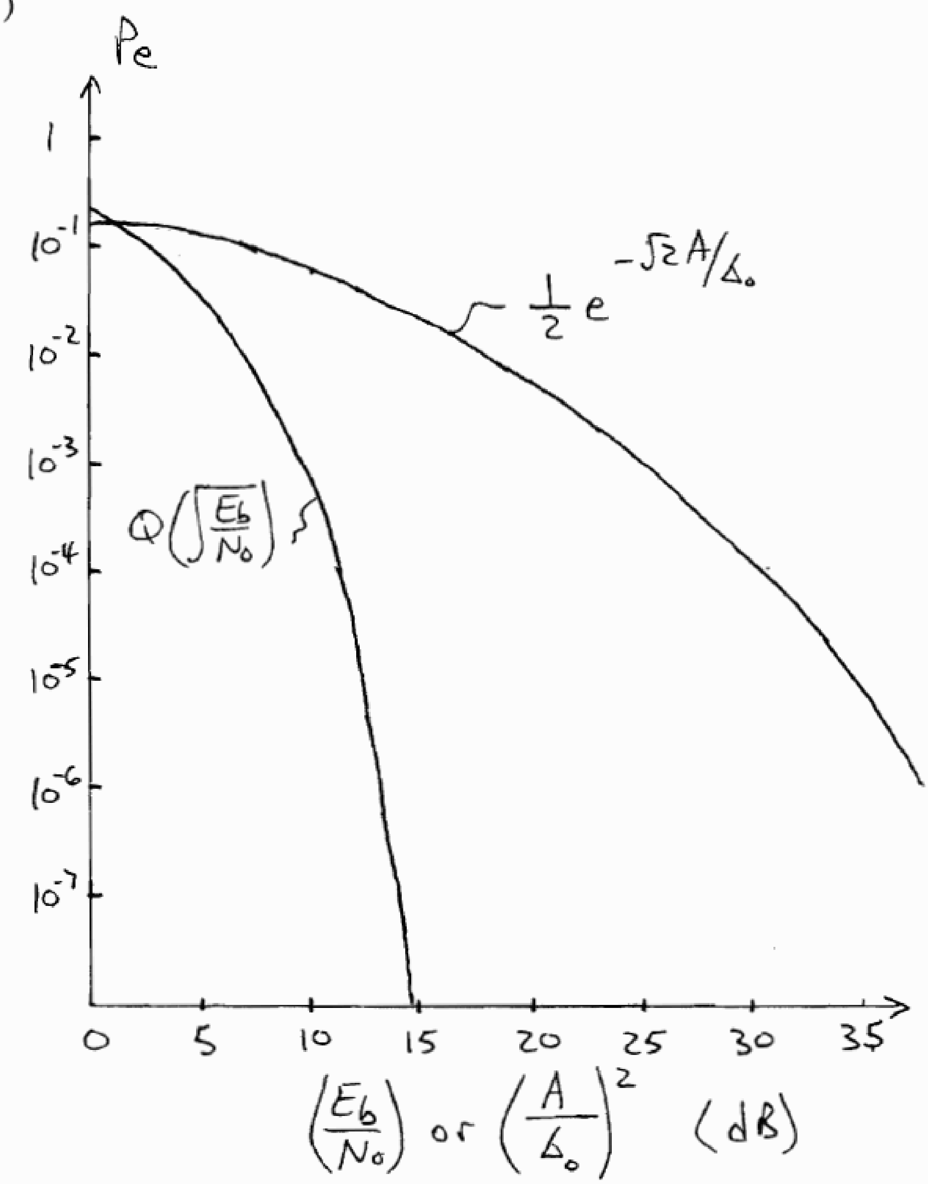
$$= \frac{1}{4}\left[\int_{-\infty}^{-\sqrt{2}A/\Delta_o} e^{x_1} dx_1 + \int_{-\infty}^{-\sqrt{2}A/\Delta_o} e^{x_2} dx_2\right]$$

7-1 (Continued)

$$P_e = \frac{1}{2}\left[\int_{-\infty}^{-\sqrt{2}A/\Delta_0} e^{x}\,dx\right] = \frac{1}{2}\, e^{x}\Big|_{-\infty}^{-\sqrt{2}A/\Delta_0}$$

$$= \frac{1}{2}\left[e^{-\sqrt{2}A/\Delta_0} - e^{-\infty}\right] = \underline{\underline{\frac{1}{2}\, e^{-\sqrt{2}A/\Delta_0}}} = P_e$$

(b.)



P_e <u>much</u> larger for Laplacian Noise.

7-4

$$\left(\frac{S}{N}\right)_{in} = \frac{\frac{E_b}{T_b}}{\left(\frac{N_0}{2}\right)(2B_{eq})} = \frac{E_b R}{N_0 B_{eq}} \Rightarrow \frac{E_b}{N_0} = \frac{B_{eq}}{R}\left(\frac{S}{N}\right)_{in}$$

Aside:

$$B_{eq} = \frac{\int_{-\infty}^{\infty} |H(f)|^2 df}{2|H(0)|^2} = \frac{\left(\frac{2K}{N_0}\right)^2 \int_{-\infty}^{\infty} |S(f)|^2 df}{2\left(\frac{2K}{N_0}\right)^2 |S(0)|^2} = \frac{\int_{-\infty}^{\infty} |S(f)|^2 df}{2|S(0)|^2}$$

Using (6-155) for MF: $H(f) = \dfrac{K S^*(f) e^{-j\omega t_0}}{N_0/2}$

$$\Rightarrow B_{eq} = \frac{T_b^2 \int_{-\infty}^{\infty} \left(\frac{\sin(\pi T_b f)}{\pi T_b f}\right)^2 df}{2 T_b^2 1^2} = \frac{1}{2\pi T_b}\int_{-\infty}^{\infty} \left(\frac{\sin x}{x}\right)^2 dx = \frac{1}{2\pi T_b}\pi = \frac{1}{2}R$$

$s(t) = \Pi\left(\frac{t}{T_b}\right) \leftrightarrow S(f) = T_b \dfrac{\sin(\pi T_b f)}{\pi T_b f}$

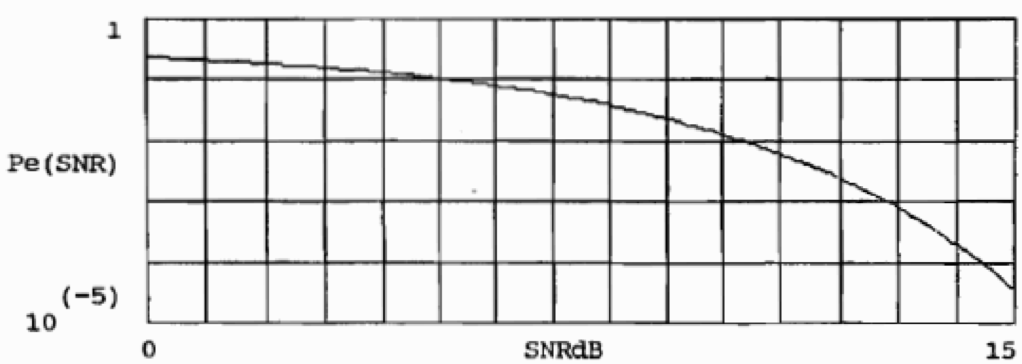
Let $x = \pi T_b f$
$dx = \pi T_b \, df$

$$\Rightarrow \frac{E_b}{N_0} = \frac{\frac{1}{2}R}{R}\left(\frac{S}{N}\right)_{in} = \frac{1}{2}\left(\frac{S}{N}\right)_{in} = \frac{E_b}{N_0}$$

Using (7-24b):

$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{1}{2}\left(\frac{S}{N}\right)_{in}}\right)$$

```
SNRdB := 0,0.1 ..15
Q(x) := 1 - cnorm(x)
SNR(SNRdB) := 10^(SNRdB/10)
Pe(SNR) := Q[√(0.5·SNR(SNRdB))]
```



7-6

(a.) For derivation of P_e, follow the same procedure as used in the solution for Prob. 7-7.

$$B_{eq} = \int_0^\infty \frac{|H(f)|^2}{|H(0)|^2}\,df \underset{\text{Let } x = f/f_0}{=} \int_0^\infty \frac{1}{1+\left(\frac{f}{f_0}\right)^4}\,df = f_0 \int_0^\infty \frac{1}{1+x^4}\,dx \underset{\text{Using Sec. A-3}}{=} \frac{f_0 \pi}{2\sqrt{2}}$$

$$s_{01} = \int_0^T s_{01}(T-\lambda)\,h(\lambda)\,d\lambda = \int_0^T A\left[\sqrt{2}\,\omega_0 e^{-(\omega_0/\sqrt{2})\lambda} \sin\left(\frac{\omega_0}{\sqrt{2}}\lambda\right)\right] d\lambda$$

or

$$s_{01} \underset{\text{Let } x = \frac{\omega_0}{\sqrt{2}}\lambda}{=} \int_0^T 2A\, e^{-x} \sin(x)\,dx \underset{\text{Using Sec. A-5}}{=} A\left[1 - e^{-\sqrt{2}\pi}\left(\sin(\sqrt{2}\pi) + \cos(\sqrt{2}\pi)\right)\right]$$

$$\Rightarrow \underline{s_{01} = 1.01447\,A}$$

$$\sigma_0^2 = \frac{N_0}{\cancel{2}}\,\cancel{2}B_{eq} = N_0 B_{eq} = \frac{N_0 \pi f_0}{2\sqrt{2}} \underset{f_0 = \frac{1}{T}}{=} \frac{N_0 \pi}{2\sqrt{2}\,T}$$

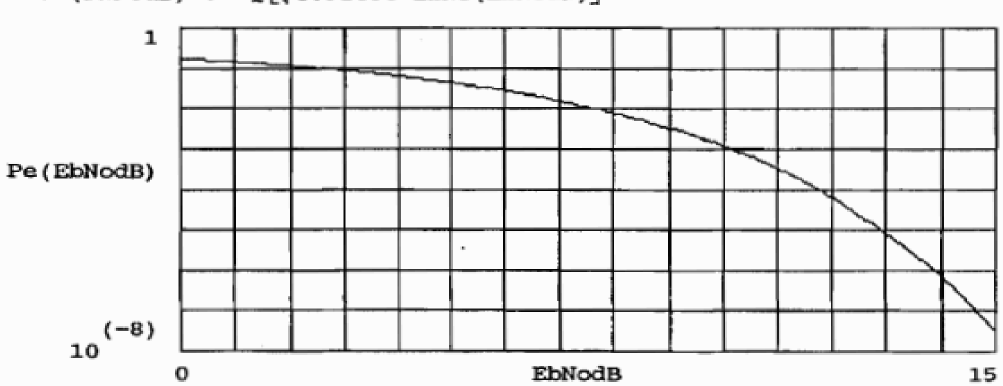
Using (7-17) where $s_{01} = -s_{02}$,

$$P_e = Q\left(\sqrt{\frac{s_{01}^2}{\sigma_0^2}}\right) = Q\left(\sqrt{\frac{(1.01447)^2 A^2}{\frac{N_0 \pi}{2\sqrt{2}\,T}}}\right) = Q\left(\sqrt{\frac{2\sqrt{2}\,(1.01447)^2 A^2 T}{\pi N_0}}\right)$$

$$\Rightarrow P_e = Q\left(\sqrt{\frac{2\sqrt{2}\,(1.01447)^2}{\pi}\left(\frac{E_b}{N_0}\right)}\right) = \underline{\underline{Q\left(\sqrt{0.92656\left(\frac{E_b}{N_0}\right)}\right)}}$$

(b.)

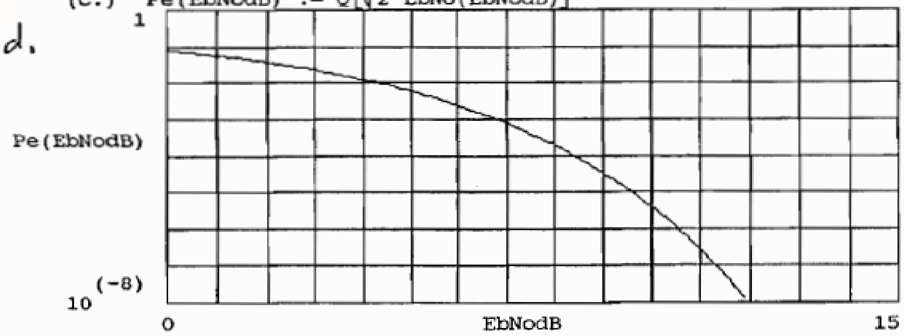
```
EbNodB := 0,0.1 ..15
Q(x) := 1 - cnorm(x)
EbNo(EbNodB) := 10^(EbNodB/10)
Pe(EbNodB) := Q[√(0.92656·EbNo(EbNodB))]
```



7-6 (Continued)

(c.) $Pe(EbNodB) := Q\left[\sqrt{2 \cdot EbNo(EbNodB)}\right]$

d.



7-9

(a.) Referring to the solution for SA 7-3,

$$P_e = Q\left(\sqrt{\frac{A^2}{4N_0B}}\right) \qquad (7\text{-}24a)$$

where $B = \frac{2}{T} = 2R$ and $E_b = \left(\frac{A^2}{2}\right)T = \frac{A^2}{2R}$

Thus, $P_e = Q\left(\sqrt{\frac{A^2}{8N_0R}}\right) = Q\left(\sqrt{\frac{A^2}{4N_0 2R}}\right) = Q\left(\sqrt{\frac{1}{4}\left(\frac{E_b}{N_0}\right)}\right)$

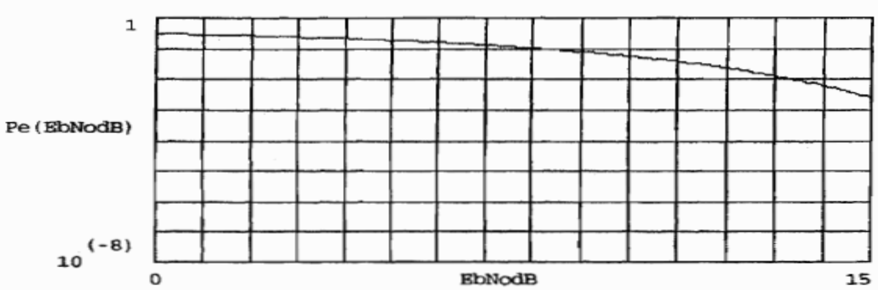
$EbNodB := 0, 0.1 \,..\, 15$

$Q(x) := 1 - \text{cnorm}(x)$

$EbNo(EbNodB) := 10^{\frac{EbNodB}{10}}$

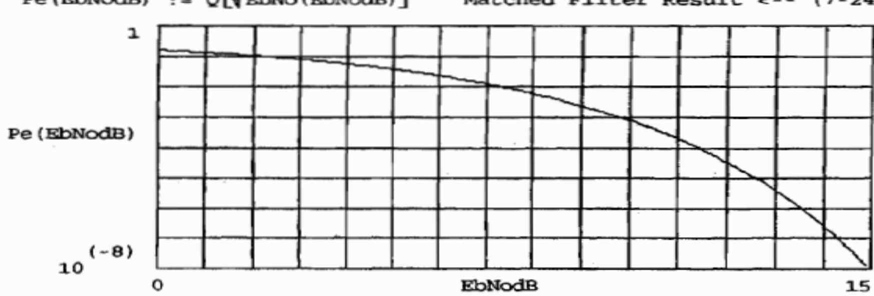
(a.)

$Pe(EbNodB) := Q\left[\sqrt{0.25 \cdot EbNo(EbNodB)}\right]$ <---- LPF with ISI Result



(b.)

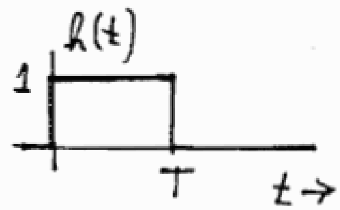
$Pe(EbNodB) := Q\left[\sqrt{EbNo(EbNodB)}\right]$ Matched Filter Result <-- (7-24b)



7-11

(a.) The impulse response is:

$$h(t) = s_{01}(T-t) = \underline{\underline{s_{01}(t)}}$$



$$H(f) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt = \int_0^T e^{-j\omega t}\, dt = \left.\frac{e^{-j\omega t}}{j\omega}\right|_0^T = \frac{e^{-j\omega T} - e^{-j(0)}}{-j\omega}$$

$$\Rightarrow H(f) = e^{-j\omega T/2}\left[\frac{e^{j\omega T/2} - e^{-j\omega T/2}}{j\,2\pi f}\right] = \underline{\underline{T e^{-j\pi f T}\left[\frac{\sin(\pi f T)}{\pi f T}\right]}}$$

See Fig. 6-17

(b.)

$$B_{eq} = \frac{\int_0^{\infty} |H(f)|^2\, df}{|H(0)|^2} = \frac{T^2 \int_0^{\infty}\left[\frac{\sin(\pi f T)}{\pi f T}\right]^2 df}{T^2}$$

$$= \int_0^{\infty}\left(\frac{\sin x}{x}\right)^2\left(\frac{1}{\pi T}\, dx\right) = \frac{1}{\pi T}\left(\frac{\pi}{2}\right) = \underline{\underline{\frac{1}{2T} = B_{eq}}}$$

Let $x = \pi f T$; $dx = \pi T\, df$

Using Sec. A-5

7-13

From (7-8)

$$P_e = P(1)\int_{-\infty}^{V_T} f(r_0|s_1)\,dr_0 + P(0)\int_{V_T}^{\infty} f(r_0|s_2)\,dr_0$$

where $r_0 = \begin{cases} A + n_0, & \text{for a binary 1 sent} \\ -A + n_0, & \text{" } 0 \text{ "} \end{cases}$

Thus

$$P_e = P(1)\int_{-\infty}^{V_T} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(r_0-A)^2}{2\sigma^2}}\,dr_0 + P(0)\int_{V_T}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(r_0+A)^2}{2\sigma^2}}\,dr_0$$

$$\boxed{\begin{array}{ll} \text{Let } \lambda_1 = -(r_0-A)/\sigma & ; \quad \lambda_2 = (r_0+A)/\sigma \\ d\lambda_1 = -\frac{1}{\sigma}\,dr_0 & \quad d\lambda_2 = \frac{1}{\sigma}\,dr_0 \end{array}}$$

$$\Rightarrow P_e = P(1)\int_{(-V_T+A)/\sigma} \frac{1}{\sqrt{2\pi}} e^{-\lambda_1^2/2}\,d\lambda_1 + P(0)\int_{(V_T+A)/\sigma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\lambda_2^2/2}\,d\lambda_2$$

or

$$P_e = P(1)\,Q\left[\frac{-V_T+A}{\sigma}\right] + P(0)\,Q\left[\frac{V_T+A}{\sigma}\right]$$

$$= P(1)\,Q\left[\sqrt{\frac{(-V_T+A)^2\,2T}{N_0}}\right] + P(0)\,Q\left[\sqrt{\frac{(V_T+A)^2\,2T}{N_0}}\right]$$

$\sigma^2 = \frac{N_0}{2T}$

To Check : Let $P(1) = P(0) = \frac{1}{2}$; $V_T = 0$

$$P_e = \frac{1}{2}\,Q\left[\sqrt{\frac{A^2\,2T}{N_0}}\right] + \frac{1}{2}\,Q\left[\sqrt{\frac{A^2\,2T}{N_0}}\right]$$

$$= Q\left[\sqrt{\frac{2A^2T}{N_0}}\right]$$; This checks with eqn. (7-26b)

7-16

Referring to Fig. 7-7

(a.) Let $s_1(t) = A\cos w_c t$, $s_2(t) = -A\cos w_c t$

$n(t) = x(t)\cos w_c t - y(t)\sin w_c t$

Coherent reference $= 2\cos(w_c t + \theta_e)$

$$\Rightarrow r_0(t) \underset{(A-11)}{=} \pm A\cos\theta_e + n_0(t) = s_{o_{1,2}}(t) + n_0(t)$$

where $n_0(t) = x(t)\cos\theta_e + y(t)\sin\theta_e$

$$\overline{n_0^2(t)} = \overline{x^2(t)}\cos^2\theta_e + 2\underbrace{\overline{x(t)y(t)}}_{0\ ((6-133k))}\cos\theta_e\sin\theta_e + \overline{y^2(t)}\sin^2\theta_e$$

$$\Rightarrow \overline{n_0^2(t)} \underset{\overline{x^2}=\overline{y^2}=2N_0B}{=} 2N_0B(\cos^2\theta_e + \sin^2\theta_e) = 2N_0B$$

Using (7-17): $H(f) = $ LPF

Corresponds to (7-36)

$$P_e = Q\left[\pm\sqrt{\frac{(s_{o1}-s_{o2})^2}{4\sigma_0^2}}\right] = Q\left[\pm\sqrt{\frac{4A^2\cos^2\theta_e}{8N_0B}}\right] = Q\left[\pm\sqrt{\frac{A^2\cos^2\theta_e}{2N_0B}}\right]$$

where $\begin{cases} + \text{ is used when } s_{o1} > s_{o2} \Rightarrow |\theta_e| < \pi/2 \\ - \text{ is used when } s_{o2} > s_{o1} \Rightarrow \frac{\pi}{2} < |\theta_e| < \pi \end{cases}$

(b.) If $H(f)$ is matched to the output of the multiplier, then

$$P_e = Q\left(\sqrt{\frac{E_d}{2N_0'}}\right) \quad \text{using (7-20)}$$

Where $N_0' = 2N_0$ and

$$E_d = \int_0^T [s_{o1}(t) - s_{o2}(t)]^2\,dt = \int_0^T 4A^2\cos^2\theta_e\,dt$$

or

$$E_d = 4A^2T\cos^2\theta_e \underset{E_b = \frac{1}{2}E_{s_1} + \frac{1}{2}E_{s_2} = \frac{1}{2}\left(\frac{1}{2}A^2T\right) + \frac{1}{2}\left(\frac{1}{2}A^2T\right) = \frac{1}{2}A^2T}{=} 8E_b\cos^2\theta_e$$

$$\Rightarrow P_e = Q\left[\pm\sqrt{2\left(\frac{E_b}{N_0}\right)\cos^2\theta_e}\right]$$

Corresponds to (7-38)

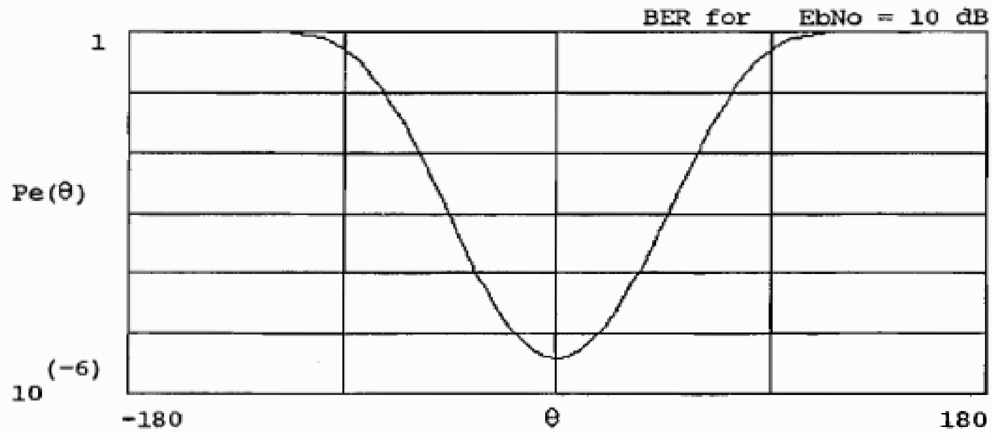
7-16 (Continued)

(c.) EbNodB := 10 θ := −180,−175 ..180 $\frac{\text{EbNodB}}{10}$

Q(x) := 1 − cnorm(x)   EbNo := 10

$$z(\theta) := \sqrt{2\ \text{EbNo}\cdot\left[\cos\left[\pi\,\frac{\theta}{180}\right]\right]^2}$$

Pe(θ) := if(|θ| ≤ 90, Q(z(θ)), Q(−z(θ)))



7-17

(a.) Overall $P_e = 10\,(P_e)_i = \underline{\underline{5\times10^{-7}}}$

(b.) when repeaters were used, the E_b/N_o at the input to each was described by:

$$P_e = Q\left[\sqrt{2\left(\frac{E_b}{N_o}\right)}\right] = 5\times10^{-8} \approx \frac{1}{\sqrt{2\pi\left(\frac{2E_b}{N_o}\right)}}\, e^{-E_b/N_o}$$

Sec. A-10

$$\Rightarrow \frac{E_b}{N_o} = \underline{14.2}$$

Now with 10 amplifiers, the Rx input consists of the BPSK signal with E_b energy/bit plus a noise level 10 times that present before (since the line from one amp to the next contributes a PSD of $N_o/2$, and there are 10 such lines). Thus $\left(\frac{E_b}{N_o}\right)' = \frac{14.2}{10} = \underline{1.42}$

$$\therefore\ P_e' = Q\left(\sqrt{2(1.42)}\right) = Q(1.69) \approx Q(1.7) = \underline{\underline{4.4\times10^{-2}}}$$

7-18

(a.) $B_T = 2700 - 300 = 2400$ Hz (Bandpass System)

The largest bit rate that can be accomodated w/o I.S.I. is (Table 7-1.)

$$\underline{\underline{B = R = 2400}} \text{ bits/sec}$$

(b.) From Table 7.1 for BPSK:

$$P_e = Q\left(\sqrt{2\left(\frac{E_b}{N_o}\right)}\right) \underset{\text{Sec. A-10}}{\approx} \frac{1}{\sqrt{4\pi\left(\frac{E_b}{N_o}\right)}} e^{-E_b/N_o}$$, for each repeater

But $\left(\frac{S}{N}\right) = \frac{P_s}{N_o B_T} \underset{B_T = R}{=} \frac{P_s}{N_o R} \underset{R = \frac{1}{T}}{=} \frac{P_s T}{N_o} = \frac{E_b}{N_o} = \frac{S}{N} = 15\text{dB}$

$$\Rightarrow \frac{E_b}{N_o} = 15\text{dB} = 31.6$$

$$\therefore P_e = \frac{1}{\sqrt{4\pi(31.6)}} e^{-31.6} = 9.26 \times 10^{-16} / \text{repeater}$$

There are $n = \frac{600 \text{ mi.}}{50 \text{ mi/rep}} = 12$ repeaters (including R_x)

$$\therefore \text{Overall } (P_e) \approx n P_e = 12(9.26 \times 10^{-16}) = \underline{\underline{1.11 \times 10^{-14}}}$$

<u>Note</u>: If there are n repeaters, there is an error at the end of the line only if there are an <u>odd</u> number of errors along the line (for the bit in question).

$$\Rightarrow P(\underline{k} \text{ errors}) = \binom{n}{k} P_e^{\,k} (1 - P_e)^{n-k} \; ; \; \binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

$$\Rightarrow \text{Overall } P_e = \sum_{\substack{k=1 \\ k \text{ odd}}}^{n} P(k \text{ errors}) \approx n P_e \quad \text{If } nP_e \ll 1$$

7-21

From Table 7-1 for FSK w/ noncoherent detection:

$$P_e = \frac{1}{2} e^{-\frac{1}{2}\left(E_b/N_{total}\right)}$$

$$N_{total} = k\left(T_o + T_{eff}\right) = k\left(T_o + (F-1)T_o\right) = kFT_o$$

$T_{eff} = (F-1)T_o$

$$= \left(1.38\times10^{-23}\right)\left(10^{6/10}\right)290$$

$$\frac{E_b}{N_{total}} = \frac{P_s T}{N_{tot}} = \frac{P_s}{N_{tot} R} = \frac{V_s^2/R_A}{kFT_o R} = 28.53$$

$R = 1/T$

$$P_e = \frac{1}{2} e^{-\frac{1}{2}(28.53)} = \underline{\underline{3.2\times10^{-7}}}$$

7-24

See Table 7-1.

(a.) For QPSK the largest R and min P_e are obtained.

$$R = 2B = 2(2700-300) = \underline{\underline{4800 \text{ b/s}}}$$

$$\frac{S}{N} = \frac{P_s}{\frac{N_o}{2}(2B)} = \frac{2P_s}{N_o R} = \frac{2P_sT}{N_o} = \frac{2E_b}{N_o}$$

$B = \frac{1}{2}R$ $\quad$ $R = \frac{1}{T}$

$$\Rightarrow \frac{E_b}{N_o} = \frac{1}{2}\left(\frac{S}{N}\right) = \frac{1}{2}\left(10^{2.5}\right) = 158.1 \Rightarrow 22\text{ dB}$$

$$P_e = Q\left(\sqrt{2\left(\frac{E_b}{N_o}\right)}\right) \ll 10^{-5} \text{ for } \frac{E_b}{N_o} = 22\text{ dB}$$

Figure 7-14.

7-24 (Continued)

(b.)

$$C = B\log_2\left(1+\frac{S}{N}\right) = 2400\log_2\left(1+10^{2.5}\right)$$

$$= \frac{2400}{\ln 2}\ln(1+316.23) = 1.99\times10^4 = \underline{\underline{19{,}900\ b/s}}$$

7-25

(a.) $R = \left(8K\ \frac{\text{samples}}{\text{sec}}\right)\left(8\ \frac{\text{bits}}{\text{sample}}\right) = 64\ K\ b/s$

$$\frac{S}{N} = \frac{P_s}{N_oB} \underset{\uparrow}{=} \frac{P_sT_b}{N_o} = \frac{E_b}{N_o} = 10^{.8} = 6.3$$

$B = R = \frac{1}{T_b} \leftarrow$ Table 7-1. for DPSK :

$$P_e = \frac{1}{2}e^{-\left(\frac{E_b}{N_o}\right)} = \frac{1}{2}e^{-6.3} = \underline{\underline{9.18\times10^{-4}}}$$

(b.) Using (7-70) with $M = 2^8 = 256$:

$$\left(\frac{S}{N}\right)_{OUT} = \frac{3M^2}{1+4(M^2-1)P_e} = \frac{3(256)^2}{1+4\left[(256)^2-1\right]9.18\times10^{-4}}$$

$$= 813.6 \Longrightarrow \underline{\underline{29.1\ dB}}$$

7-27

Using (7-89),

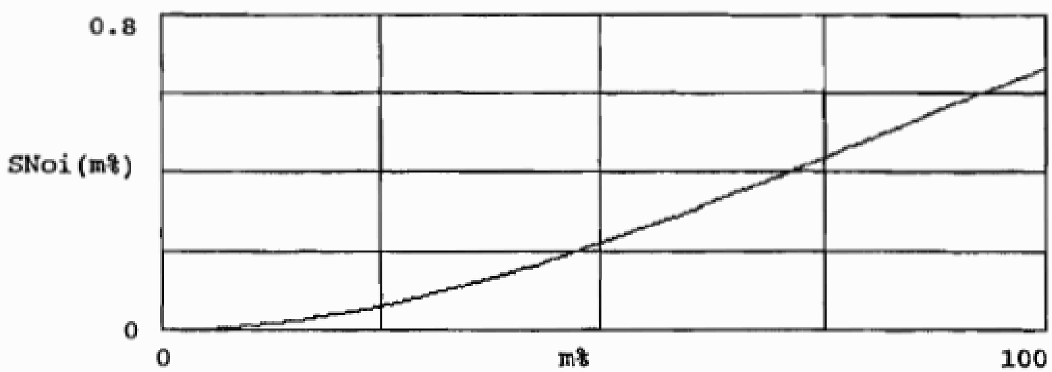
$$SN_{oi} \triangleq \frac{(S/N)_{out}}{(S/N)_{in}} = \frac{2\overline{m^2}}{1+\overline{m^2}}$$

Let $m(t) = \left(\frac{m\%}{100}\right)\cos w_m t \Rightarrow \overline{m^2} = \frac{1}{2}\left(\frac{m\%}{100}\right)^2$

$$\Rightarrow SN_{oi} = \frac{\left(\frac{m\%}{100}\right)^2}{1+\frac{1}{2}\left(\frac{m\%}{100}\right)^2}$$

m% := 0,5 ..100

$$SNoi(m\%) := \frac{\left[\frac{m\%}{100}\right]^2}{1+0.5\left[\frac{m\%}{100}\right]^2}$$



7-28

$$m(t) = 0.4\sin w_m t \Rightarrow \overline{m^2} = \frac{(0.4)^2}{2} = 0.08$$

For AM, with product detector, use (7-90).

$$\frac{(S/N)_{out}}{(S/N)_{base}} = \frac{\overline{m^2}}{1+\overline{m^2}} = \frac{0.08}{1.08} = \underline{0.0741 \Rightarrow -11.3\text{dB}}$$

Also get same result for env. det when (S/N) is large.

For DSB-SC, use (7-98).

$$\frac{(S/N)_{out}}{(S/N)_{base}} = \underline{1 \Rightarrow 0\text{dB}} \Rightarrow \underline{\text{The AM system is inferior by 11.3dB}}$$

7-31

Using Carson's Rule:

$B_{IF} = 2(\beta_f+1)B \Rightarrow 25\,KHz = 2(\beta_f+1)5\,KHz \Rightarrow \beta_f = 1.5$

$f_1 = 2.1\,KHz \;;\; \frac{B}{f_1} = \frac{5}{2.1} \not\gg 1 \;\therefore$ (7-139) is not valid

Eqn. (7-124a) $\quad s_o(t) = \frac{K D_f}{2\pi} m(t) = \frac{K B \beta_f}{V_p} m(t)$

$\overline{s_o^2(t)} = K^2 B^2 \beta_f^2 \overline{\left(\frac{m}{V_p}\right)^2} \;;\; \overline{\left(\frac{m}{V_p}\right)^2} = \frac{1}{2}$ for sinusoid

Eqn. (7-136) $\overline{[\tilde{n}_o(t)]^2} = 2\left(\frac{K}{A_c}\right)^2 N_o f_1^3\left[\frac{B}{f_1} - \tan^{-1}\left(\frac{B}{f_1}\right)\right]$

$$\therefore \left(\frac{S}{N}\right)_o = \frac{K^2 B^2 \beta_f^2 \overline{\left(\frac{m}{V_p}\right)^2}}{2\left(\frac{K}{A_c}\right)^2 N_o f_1^3\left[\frac{B}{f_1} - \tan^{-1}\left(\frac{B}{f_1}\right)\right]}$$

Eqn. (7-128) $\left(\frac{S}{N}\right)_{in} = \frac{A_c^2}{4N_o(\beta_f+1)B}$

$$\frac{(S/N)_o}{(S/N)_{in}} = \frac{2\left(\frac{B}{f_1}\right)^3 \beta_f^2(\beta_f+1)\overline{\left(\frac{m}{V_p}\right)^2}}{\left[\frac{B}{f_1} - \tan^{-1}\left(\frac{B}{f_1}\right)\right]} = \frac{2(13.5)(2.25)(2.5)\frac{1}{2}}{1.21} = 62.9$$

$N_{total} = KFT_o \;;\; F = 10^{1.2} = 15.8$

$$\left(\frac{S}{N}\right)_{in} = \frac{P_s}{\frac{N_{tot}}{2}(2B_{IF})} = \frac{P_s}{KFT_oB_{IF}} = \frac{P_s}{1.57\times10^{-15}}$$

$$\Rightarrow P_s = \left(\frac{S}{N}\right)_o\left(\frac{1}{62.9}\right)(1.57\times10^{-15})$$

$$= 10^{3.5}\left(\frac{1}{62.9}\right)(1.57\times10^{-15}) = 7.89\times10^{-14}\ \text{W}$$

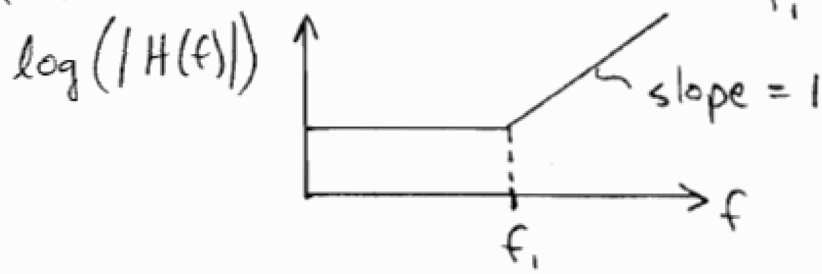
$$10\log_{10}\left(\frac{7.89\times10^{-14}}{10^{-3}}\right) = \underline{\underline{-101\ \text{dBm}}} = P_{s\,min}$$

7-35

$$H(f) = [1 + j f/f_1] \text{ for preemphasis}$$

(a.)

$f_1 = 2.1$ kHz



At $f = 15$ kHz the gain is:

$$|H(f)| = \left|1 + j\left(\frac{15}{2.1}\right)\right| = \sqrt{1 + \left(\frac{15}{2.1}\right)^2} = 7.21$$

At $f = 1$ kHz:

$$|H(f)| = \left|1 + j\left(\frac{1}{2.1}\right)\right| = \sqrt{1 + \left(\frac{1}{2.1}\right)^2} = 1.10$$

$$\Delta F = 75\text{ kHz}\left(\frac{7.21}{1.10}\right) = \underline{\underline{488\text{ kHz}}}$$

$$\%\text{ mod} = \frac{488}{75}(100) = \underline{\underline{651\%\text{ mod}}}$$

(b.) The amplitudes of the high frequency audio components are much smaller than those of the low frequency components. For example, if the components at 15 kHz are more than $20\log_{10}\left(\frac{7.21}{1.1}\right) = 16.3$ dB below the 1 kHz components, there is no problem.