Chapter 6

$\boxed{6\text{-}2.}$ (a.)

$$\overline{x(t)} = \int_0^{\pi/2} \frac{2}{\pi} \cdot A \cos(\omega_0 t + \theta)\, d\theta$$

$$= \frac{2A}{\pi} \sin(\omega_0 t + \theta) \Big|_0^{\pi/2}$$

$$= \frac{2A}{\pi} \left[\sin(\omega_0 t + \pi/2) - \sin \omega_0 t \right]$$

$$= \frac{2A}{\pi} \left[\cos \omega_0 t - \sin \omega_0 t \right]$$

$$= \frac{2\sqrt{2} A}{\pi} \left[\cos\left(\frac{\pi}{4}\right) \cos \omega_0 t - \sin\left(\frac{\pi}{4}\right) \sin \omega_0 t \right]$$

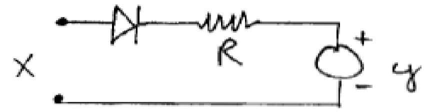
$$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} = \sin \frac{\pi}{4}$$

$$= \frac{2\sqrt{2} A}{\pi} \cos\left(\omega_0 t + \frac{\pi}{4}\right)$$

(b.) $\overline{x(t)}$ is a function of time

$\therefore$ $x(t)$ is not stationary.

6-4.



Let y be the current thru the meter produced by the voltage x.

For $x = V_p \cos\omega_0 t$:

$$\langle y \rangle = \frac{1}{T_0}\int_{-T_0/4}^{T_0/4} \frac{V_p}{R}\cos\omega_0 t \, dt = \frac{2V_p}{T_0 R}\int_0^{T_0/4} \cos\omega_0 t \, dt$$

$$= \frac{2V_p}{R T_0} \left. \frac{\sin\omega_0 t}{\omega_0} \right|_0^{T_0/4} = \frac{V_p}{R\pi} = \frac{\sqrt{2}}{R\pi} V_{rms}$$

$$\omega_0 = \frac{2\pi}{T_0}$$

$$V_{meter} = V_{rms} = \frac{R\pi}{\sqrt{2}}\langle y \rangle$$

Let x be a zero-mean, ergodic Gaussian noise voltage.

$$\langle y \rangle = \frac{1}{R}\int_0^{\infty} x\, p(x)\, dx = \frac{1}{R\sqrt{2\pi}\,\Delta}\int_0^{\infty} x\, e^{-x^2/2\Delta^2} dx$$

where Δ = rms value of x

Let $z = \frac{x^2}{2\Delta^2}$; $dz = \frac{x}{\Delta^2}\, dx$

$$\langle y \rangle = \frac{\Delta}{R\sqrt{2\pi}}\int_0^{\infty} e^{-x^2/2\Delta^2}\left(\frac{x}{\Delta^2}\right) dx$$

$$= \frac{\Delta}{R\sqrt{2\pi}}\int_0^{\infty} e^{-z}\, dz = \left. \frac{-\Delta\, e^{-z}}{R\sqrt{2\pi}} \right|_0^{\infty}$$

$$= \frac{-\Delta}{R\sqrt{2\pi}}\left[e^{-\infty} - e^{-0}\right] = \frac{\Delta}{R\sqrt{2\pi}} = \langle y \rangle$$

$$V_{meter} = \frac{R\pi}{\sqrt{2}}\langle y \rangle = \frac{R\pi}{\sqrt{2}}\left[\frac{\Delta}{R\sqrt{2\pi}}\right] = \frac{\sqrt{\pi}}{2}\Delta$$

$$\therefore \quad \Delta = V_{meter}\left[\frac{2}{\sqrt{\pi}}\right]$$

6-8. Ergodicity $\Rightarrow \langle [\;] \rangle = \overline{[\;]}$

(a.) $P = \overline{n^2(t)} = \overline{[n_1(t) + n_2(t)]^2}$

$= \overline{n_1^2(t)} + 2\,\overline{n_1(t)n_2(t)} + \overline{n_2^2(t)}$, where $\overline{n_1(t)n_2(t)} \to 0$ (orthogonal)

$= \overline{n_1^2(t)} + \overline{n_2^2(t)}$

$= 5 + 10 = \underline{\underline{15 \text{ Watts}}}$

(b.) $P = \overline{n^2(t)} = \overline{n_1^2(t)} + 2\,\overline{n_1(t)n_2(t)} + \overline{n_2^2(t)}$, where $2\,\overline{n_1(t)n_2(t)} \to 2\,\overline{n_1(t)}\;\overline{n_2(t)}$ (uncorrelated)

$P = 5 + 2(-2)(1) + 10 = \underline{\underline{11 \text{ Watts}}}$

(c.) $P = \overline{n^2(t)} = \overline{[n_1(t) + n_2(t)]^2}$

$= \overline{n_1^2(t)} + 2\,\overline{n_1(t)n_2(t)} + \overline{n_2^2(t)}$, where $2\,\overline{n_1(t)n_2(t)} \to 2R_{n_1 n_2}(0)$

$= 5 + 2(2) + 10 = \underline{\underline{19 \text{ Watts}}}$

$\boxed{6\text{-}10.}$ (a.) $\sin \omega_0 \tau$ ① $R(\tau) \neq R(-\tau)$ ×

NO

(b.) $\left(\dfrac{\sin \omega_0 \tau}{\omega_0 \tau}\right)$ ① $R(\tau) = R(-\tau)$

② $R(0) \geq |R(\tau)|$

YES

③ $\mathcal{F}\left[\dfrac{\sin \omega_0 \tau}{\omega_0 \tau}\right] =$ Non-negative rectangle

(c.) $\cos \omega_0 \tau + \delta(\tau)$ ① $R(\tau) = R(-\tau)$

YES

② $R(0) \geq |R(\tau)|$

③ $\mathcal{F}\left[\cos \omega_0 \tau + \delta(\tau)\right] = \frac{1}{2}\delta(f-f_0) + \frac{1}{2}\delta(f+f_0) + 1$

≥ 0

(d.) $e^{-a|\tau|}$ where $\underline{a < 0}$ ① $R(\tau) = R(-\tau)$

NO

② $R(0) \ngeq |R(\tau)|$ ×

6-12. $R_x(\tau) = 4e^{-\tau^2} + 3$

(a.)

$$\mathcal{P}_x(f) = \mathcal{F}[R_x(\tau)] = \mathcal{F}[4e^{-\tau^2}] + \mathcal{F}[3] = \mathcal{F}\left[4e^{-\pi\left(\frac{\tau}{\sqrt{\pi}}\right)^2}\right] + \mathcal{F}[3]$$

$$\Rightarrow \mathcal{P}_x(f) \underset{\text{Using Table 2-2}}{=} 4\sqrt{\pi}\, e^{-\pi(f\sqrt{\pi})^2} + 3\delta(f) = \underline{\underline{4\sqrt{\pi}\, e^{-(\pi f)^2} + 3\delta(f)}}$$

(b.) This is a low-pass spectrum. $\Rightarrow$ Use (6-97) and (6-98).

$$\overline{f^2} = \frac{\int_{-\infty}^{\infty} f^2 \mathcal{P}_x(f)\,df}{\int_{-\infty}^{\infty} \mathcal{P}_x(\lambda)\,d\lambda} = \frac{4\int_{-\infty}^{\infty} f^2 \frac{1}{\sqrt{2\pi}\left(\frac{1}{\sqrt{2}\pi}\right)} e^{-f^2/2\left(\frac{1}{\sqrt{2}\pi}\right)^2} df + \underbrace{3\int_{-\infty}^{\infty} f^2 \delta(f)\,df}_{0}}{4\left(\underbrace{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\left(\frac{1}{\sqrt{2}\pi}\right)} e^{-f^2/2\left(\frac{1}{\sqrt{2}\pi}\right)^2} df}_{1}\right) + 3\underbrace{\int_{-\infty}^{\infty} \delta(f)\,df}_{1}}$$

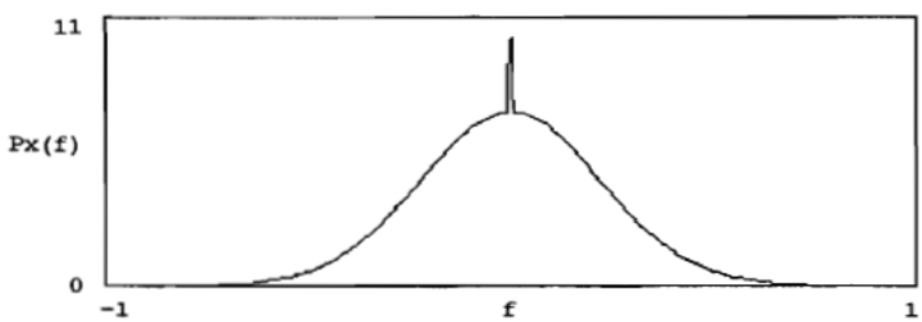
$$\Rightarrow \overline{f^2} = \frac{4\left(\frac{1}{\sqrt{2}\pi}\right)^2}{4+3} = \frac{\cancel{4}^{\,2}}{7}\,\frac{1}{\cancel{2}\pi^2} = \frac{2}{7\pi^2}$$

Thus, $B_{rms} = \sqrt{\overline{f^2}} = \sqrt{\frac{2}{7\pi^2}} = \sqrt{\frac{2}{7}}\,\frac{1}{\pi} = \underline{\underline{0.170\ \text{Hz}}}$

$\delta(0)$ cannot be plotted since it is infinity. Consequently, following the usual convention in EE, plot the WEIGHT of $\delta(f)$ instead at f=0.

f := −1,−0.99 ..1 $\qquad$ δ(f) := if(f ≈ 0,1,0)

$Px(f) := 4\sqrt{\pi}\cdot e^{-\left[(\pi\cdot f)^2\right]} + 3\cdot\delta(f)$ $\qquad$ Px(0) = 10.09



$Brms := \sqrt{\frac{2}{7}}\cdot\left[\frac{1}{\pi}\right]$ $\qquad$ Brms = 0.17

$\boxed{6\text{-}17.}$

(a.) $X_{rms} = \sqrt{\overline{x^2(t)}}$

$$\overline{x^2(t)} = R_x(0) = \int_{-\infty}^{\infty} \mathcal{P}_x(f)\,df$$

$$= \int_{-B}^{0} \frac{1}{B}(B+f)\,df + \int_{0}^{B} \frac{1}{B}(B-f)\,df$$

$$= \frac{1}{B}\left\{ \left(Bf + \frac{f^2}{2}\right)\Big|_{-B}^{0} + \left(Bf - \frac{f^2}{2}\right)\Big|_{0}^{B} \right\}$$

$$= \frac{1}{B}\left\{ -\left(-B^2 + \frac{B^2}{2}\right) + \left(B^2 - \frac{B^2}{2}\right) \right\} = \frac{1}{B}\left[2B^2 - B^2\right]$$

$$= B \implies \underline{\underline{X_{rms} = \sqrt{B}}}$$

(b.) $\mathcal{P}_x(f) = \frac{1}{\sqrt{B}}\Pi\left(\frac{f}{B}\right) * \frac{1}{\sqrt{B}}\Pi\left(\frac{f}{B}\right)$

$$R_x(\tau) = \mathcal{F}^{-1}\left[\mathcal{P}_x(f)\right] = \frac{1}{B}\left\{\mathcal{F}^{-1}\left[\Pi\left(\frac{f}{B}\right)\right]\right\}^2$$

Table 2-1 · Multiplication property of $\mathcal{F}[\,]$

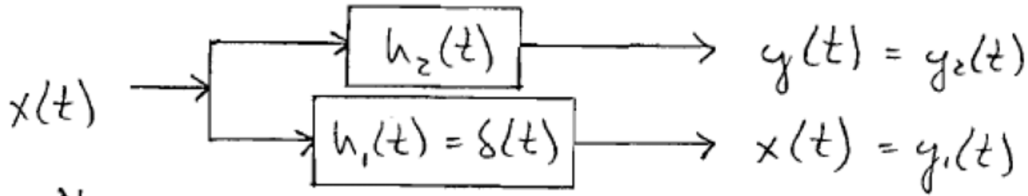
$$R_x(\tau) = \frac{1}{B}\left\{\mathcal{F}^{-1}\left[\Pi\left(\frac{f}{B}\right)\right]\right\}^2$$

Table 2-2

$$= \frac{1}{B}\left\{ B\,\frac{\sin(\pi B\tau)}{\pi B\tau} \right\}^2$$

$$\underline{\underline{R_x(\tau) = B\left[\frac{\sin(\pi B\tau)}{\pi B\tau}\right]^2}}$$

6-20.



$$\mathcal{P}_x(f) = \frac{N_0}{2} \Rightarrow R_x(\tau) = \frac{N_0}{2}\,\delta(\tau)$$

(a.) From eqn. (6-86 b.)

$$R_{y_1 y_2}(\tau) = h_1(-\tau) * h_2(\tau) * R_{x_1 x_2}(\tau)$$

but $\quad h_1(-\tau) = \delta(-\tau) = \delta(\tau)$

$$\Rightarrow R_{xy}(\tau) = \delta(\tau) * h_2(\tau) * R_x(\tau)$$

$$= \delta(\tau) * \left[h_2(\tau) * \frac{N_0}{2}\,\delta(\tau)\right]$$

$$= \delta(\tau) * \frac{N_0}{2}\,h_2(\tau) = \frac{N_0}{2}\,h_2(\tau)$$

$$\Rightarrow \underline{\underline{h_2(\tau) = R_{xy}(\tau)\left[\frac{2}{N_0}\right]}}$$

(b.) From (a.) : $R_{xy}(\tau) = \frac{N_0}{2}\,h_2(\tau)$

$$\mathcal{P}_{xy}(f) = \mathcal{F}\left[R_{xy}(\tau)\right] = \mathcal{F}\left[\frac{N_0}{2}\,h_2(\tau)\right] = \frac{N_0}{2}\,H_2(f)$$

$$\Rightarrow \underline{\underline{H_2(f) = \left[\frac{2}{N_0}\right]\mathcal{P}_{xy}(f)}}$$

6-22. (a.) $\mathcal{P}_y(f) = |H(f)|^2 \mathcal{P}_n(f) = \left|\frac{K}{j2\pi f}\right|^2 \frac{N_0}{2}$

$$\mathcal{P}_y(f) = \frac{N_0 K^2}{8\pi^2 f^2}$$

(b.) $y_{rms}^2 = R_y(0) = \int_{-\infty}^{\infty} \mathcal{P}_y(f)\, df$

$$= \int_{-\infty}^{\infty} \frac{N_0 K^2}{8\pi^2 f^2} = 2\int_0^{\infty} \left[\frac{N_0 K^2}{8\pi^2}\right] \frac{1}{f^2}\, df$$

$$= \frac{N_0 K^2}{4\pi^2}\left[\frac{-1}{f}\right]\Bigg|_0^{\infty} = \frac{N_0 K^2}{4\pi^2}\left[\frac{-1}{\infty} + \frac{1}{0}\right] = \infty$$

A practical integrator will have a large (i.e. finite) output.

6-25. From (6-95): $\left(\frac{S}{N}\right)_{out} = \frac{2A_0^2 RC}{N_0\left[1 + (2\pi f_0 RC)^2\right]}$

Let $RC = \tau$, $2\pi f_0 = \omega_0$

$$\Rightarrow \left(\frac{S}{N}\right)_{out} = \frac{2A_0^2 \tau}{N_0\left[1 + (\omega_0 \tau)^2\right]}$$

For $\max\left[\left(\frac{S}{N}\right)_{out}\right]$, set $\frac{d\left[\left(\frac{S}{N}\right)_{out}\right]}{d\tau} = 0$

$$\Rightarrow \frac{d\left[\left(\frac{S}{N}\right)_{out}\right]}{d\tau} = \frac{\left[1 + (\omega_0\tau)^2\right] 2A_0^2 - 2A_0^2 \tau (2)(\omega_0 \tau)\omega_0}{N_0\left[1 + (\omega_0\tau)^2\right]^2}$$

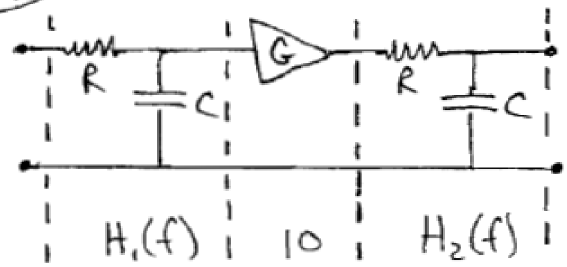
Set numerator $= 0 \Rightarrow 2A_0^2\left[1 + \omega_0^2\tau^2 - 2\tau\omega_0^2\tau\right] = 0$

$\Rightarrow \left[1 - \omega_0^2\tau^2\right] = 0 \Rightarrow \omega_0^2\tau^2 = 1 \Rightarrow \tau^2 = \frac{1}{\omega_0^2} \Rightarrow \tau = \frac{1}{\omega_0}$

Thus, $RC = \frac{1}{2\pi f_0}$ for $\max\left(\frac{S}{N}\right)_{out}$

eqn.(6-102) $\Rightarrow f_0$

$\boxed{6\text{-}30.}$ (a.)



$$H_1(f) = H_2(f) = \frac{1}{1+j\,f/f_0}$$

where $f_0 = \frac{1}{2\pi RC}$, and $G = 10$

$$H(f) = H_1(f)[10]\,H_2(f) = \underline{\underline{\frac{10}{[1+j\,f/f_0]^2}}}$$

$$|H(f)| = \frac{10}{\left|1+j\,2f/f_0 - (f/f_0)^2\right|} = \frac{10}{\sqrt{[1-(f/f_0)^2]^2 + (2f/f_0)^2}}$$

Using a programmable calculator, find the value of $f = f_c$, such that :

$$|H(f_c)| = \frac{10}{\sqrt{2}} \Longrightarrow \underline{\underline{f_c = 0.690\,f_0\ ;\ f_0 = \frac{1}{2\pi RC}}}$$

$\boxed{6\text{-}33.}$ (a.) x_1 & y_2 uncorrelated $\overset{\text{prop. 3}}{\Longrightarrow}$ Independent

when $R_{xy}(\tau) = \overline{x(t_1)\,y(t_2)} = 10\sin(2\pi\tau) = \underline{0} = \cancel{\bar{x}_1}^{0}\,\cancel{\bar{y}_2}^{0}$

$\Longrightarrow \quad 2\pi\tau = \pm n\pi \Rightarrow$ <u>These r.v.'s are independent</u>

<u>only when $t_2 - t_1 = \tau = \pm \frac{n}{2}$; $n = 0, 1, \ldots$</u>

6-33. Cont'd (b.)

$$10\sin(2\pi\tau) = 10\sin[2\pi(t_2-t_1)] = \overline{x(t_1)y(t_2)}$$

This cannot be expressed as $\overline{x(t_1)}\ \overline{y(t_2)}$

$\therefore$ $x(t)$ and $y(t)$ are <u>not indep.</u>

$\boxed{6\text{-}35.}$ (a.) Evaluate $\overline{x^2(t)} = R_x(0)$

$$\overline{x^2(t)} = \overline{A_o^2\cos^2(\omega_o t+\theta)} = \frac{A_o^2}{2}\left[1 + \overline{\cos(2\omega_o t+2\theta)}\right]$$

$$= \frac{A_o^2}{2} + \frac{A_o^2}{2}\int_0^{\pi/2}\cos(2\omega_o t+2\theta)\,\frac{2}{\pi}\,d\theta$$

$$= \frac{A_o^2}{2} + \frac{A_o^2}{2}\,\frac{\sin(2\omega_o t+2\theta)}{2}\left(\frac{2}{\pi}\right)\Bigg|_0^{\pi/2}$$

$$= \frac{A_o^2}{2} + \frac{A_o^2}{2\pi}\left[\sin(2\omega_o t+\pi) - \sin(2\omega_o t)\right]$$

$$= \frac{A_o^2}{2} + \frac{A_o^2}{2\pi}\left[-2\sin(2\omega_o t)\right]$$

Using Sec. A-1

This is a function of t $\therefore$ $x(t)$ <u>not W.S.S.</u>

6-35. Cont'd (b.)

Eqn. (6-42) $\mathcal{P}_x(f) = \lim_{T\to\infty}\left[\dfrac{\overline{|X_T(f)|^2}}{T}\right]$

where $X_T(f) = \int_{-T/2}^{T/2} x(t)\, e^{-j\omega t}\, dt$

$$= \int_{-T/2}^{T/2} A_o \cos(\omega_o t+\theta)\, e^{-j\omega t}\, dt$$

$$= A_o \int_{-T/2}^{T/2} \frac{e^{j(\omega_o t+\theta)} + e^{-j(\omega_o t+\theta)}}{2}\, e^{-j\omega t}\, dt$$

$$= \frac{A_o}{2} e^{j\theta} \int_{-T/2}^{T/2} e^{j(\omega_o-\omega)t}\, dt + \frac{A_o}{2} e^{-j\theta} \int_{-T/2}^{T/2} e^{-j(\omega_o+\omega)t}\, dt$$

$$= \frac{A_o}{2} e^{j\theta} \left.\frac{e^{j(\omega_o-\omega)t}}{j(\omega_o-\omega)}\right|_{-T/2}^{T/2} + \frac{A_o}{2} e^{-j\theta} \left.\frac{e^{-j(\omega_o+\omega)t}}{-j(\omega_o+\omega)}\right|_{-T/2}^{T/2}$$

$$= A_o\left[e^{j\theta}\,\frac{e^{j(\omega_o-\omega)T/2} - e^{-j(\omega_o-\omega)T/2}}{2j\,(\omega_o-\omega)} + e^{-j\theta}\,\frac{e^{j(\omega_o+\omega)T/2} - e^{-j(\omega_o+\omega)T/2}}{2j\,(\omega_o+\omega)}\right]$$

$$= A_o\, e^{j\theta}\,\frac{\sin(\omega_o-\omega)T/2}{(\omega_o-\omega)} + A_o\, e^{-j\theta}\,\frac{\sin(\omega_o+\omega)T/2}{(\omega_o+\omega)}$$

Let $x_1 = (\omega_o-\omega)T/2$ and $x_2 = (\omega_o+\omega)T/2$

$$= \frac{A_o T}{2}\left[e^{j\theta}\,\frac{\sin x_1}{x_1} + e^{-j\theta}\,\frac{\sin x_2}{x_2}\right]$$

$$\frac{|X_T(f)|^2}{T} = \frac{X_T(f)\, X_T^*(f)}{T} =$$

$$\frac{\left(\frac{A_o T}{2}\right)^2}{T}\left[e^{j\theta}\frac{\sin x_1}{x_1} + e^{-j\theta}\frac{\sin x_2}{x_2}\right]\left[e^{-j\theta}\frac{\sin x_1}{x_1} + e^{j\theta}\frac{\sin x_2}{x_2}\right]$$

6-35.(b.) Cont'd

$$\frac{\overline{|X_T(f)|^2}}{T} = \frac{A_o^2 T}{4}\left[\left(\frac{\sin x_1}{x_1}\right)^2 + \overline{e^{j2\theta}}\left(\frac{\sin x_1}{x_1}\right)\left(\frac{\sin x_2}{x_2}\right) + \overline{e^{-j2\theta}}\left(\frac{\sin x_1}{x_1}\right)\left(\frac{\sin x_2}{x_2}\right) + \left(\frac{\sin x_2}{x_2}\right)^2\right]$$

Aside:

$$\overline{e^{j2\theta}} = \int_0^{\pi/2} e^{j2\theta}\cdot\frac{2}{\pi}\,d\theta = j2/\pi$$

$$\overline{e^{-j2\theta}} = \int_0^{\pi/2} e^{-j2\theta}\cdot\frac{2}{\pi}\,d\theta = -j2/\pi$$

$$\frac{\overline{|X_T(f)|^2}}{T} = \frac{A_o^2}{4}\left[\frac{\overset{a}{T\pi}}{\pi}\left(\frac{\sin \pi T\overbrace{(f-f_o)}^{x}}{\pi T(f-f_o)}\right)^2 + \frac{T\pi}{\pi}\left(\frac{\sin \pi T(f+f_o)}{\pi T\underbrace{(f+f_o)}_{x'}}\right)^2\right]$$

From Sec. A-8

$$\delta(x) = \lim_{a\to\infty}\left[\frac{a}{\pi}\left(\frac{\sin ax}{ax}\right)^2\right]$$

$$\Rightarrow \quad \underline{\underline{\mathcal{P}_x(f) = \frac{A_o^2}{4}\left[\delta(f-f_o) + \delta(f+f_o)\right]}}$$

(c.) $\overline{x(t)} = A_o\,\overline{\cos(\omega_o t+\theta)} = A_o\cdot 0 = \underline{0}$

$$R_x(\tau) = \overline{x(t)\,x(t+\tau)}$$

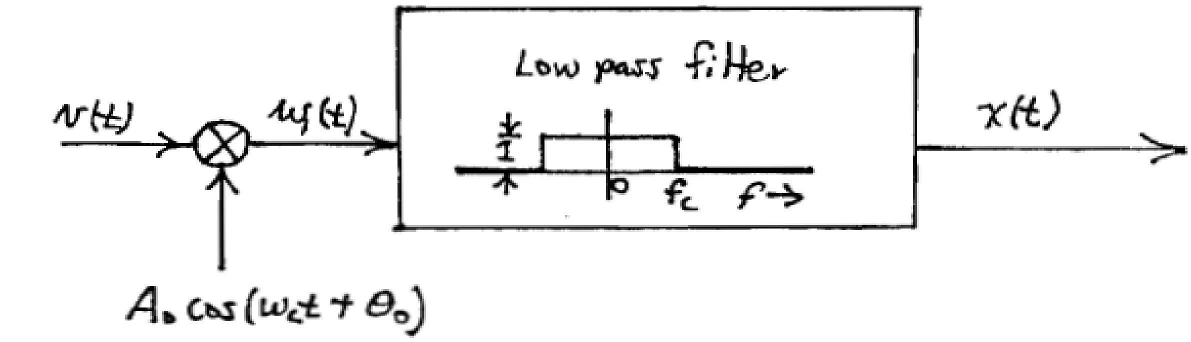
$$= A_o^2\,\overline{\cos(\omega_o t+\theta)\cos(\omega_o t+\omega_o\tau+\theta)}$$

$$= \frac{A_o^2}{2}\cos\omega_o\tau + \frac{A_o^2}{2}\underbrace{\overline{\cos(2\omega_o t+\omega_o\tau+2\theta)}}_{\to 0}$$

$$= \frac{A_o^2}{2}\cos\omega_o\tau \quad ; \text{ not a function of } t$$

$\therefore$ $\underline{\underline{x(t) \text{ is W.S.S.}}}$

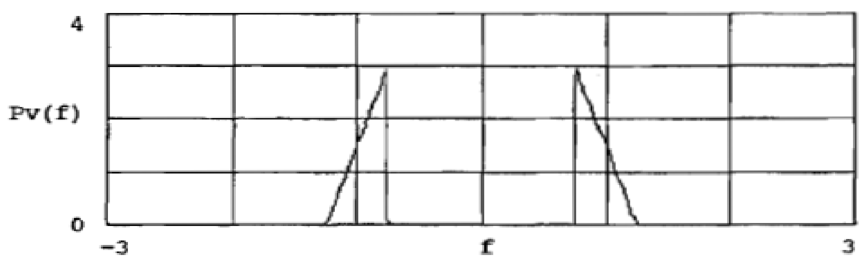
$\boxed{6\text{-}37.}$



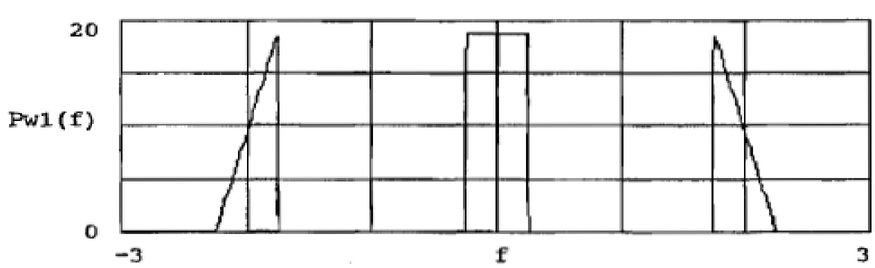
Formulas used in the solution for this problem are derived (essentially) in the textbook. See (6-141) and formulas that follow for Pw1(f) and Px(f) where the 2 in (6-141) is replaced by $(Ao)^2/2$.

$f := -3, -2.98 \,..\, 3 \qquad Ao := 5 \qquad fc := 1$

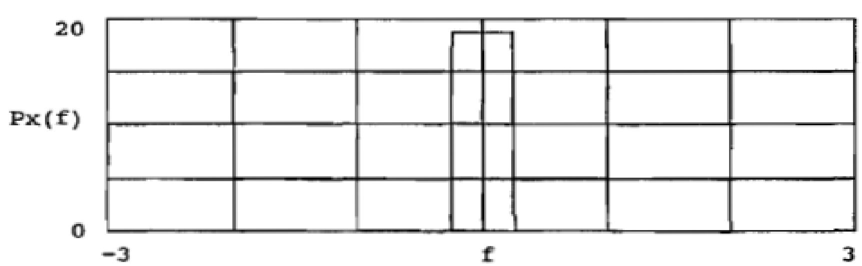
$$Pv(f) := \left[f + \frac{5}{4}\right]\cdot 6\cdot\left[\Phi\left[f + \frac{5}{4}\right] - \Phi\left[f + \frac{3}{4}\right]\right] - \left[f - \frac{5}{4}\right]\cdot 6\cdot\left[\Phi\left[f - \frac{3}{4}\right] - \Phi\left[f - \frac{5}{4}\right]\right]$$



$$Pw1(f) := \frac{Ao^2}{4}\cdot(Pv(f - fc) + Pv(f + fc))$$



$Px(f) := \text{if}(|f| < fc, Pw1(f), 0)$



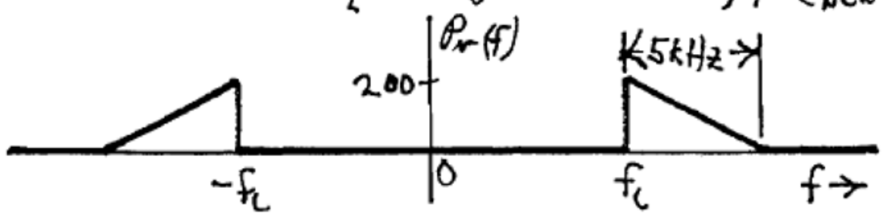
$\boxed{6\text{-}39.}$ (a.) $g(t) = 10\left[x(t) + j\hat{x}(t)\right]$

$$\Rightarrow G(f) = 10\left\{X(f) + j\begin{bmatrix} -jX(f), & f>0 \\ jX(f), & f<0 \end{bmatrix}\right\} = \begin{Bmatrix} 20X(f), & f>0 \\ 0, & f<0 \end{Bmatrix}$$

$$\Rightarrow \mathcal{P}_v(f) \underset{(6\text{-}133d)}{=} \frac{1}{4}\left[\mathcal{P}_g(f-f_c) + \mathcal{P}_g(-f-f_c)\right] = \frac{1}{4}\begin{Bmatrix} 20^2\left[\mathcal{P}_x(f-f_c) + \mathcal{P}_x(f+f_c)\right], & |f|>f_c \\ 0, & f \text{ elsewhere} \end{Bmatrix}$$

Thus,

$$\mathcal{P}_v(f) = 100\begin{Bmatrix} \left[\mathcal{P}_x(f-f_c) + \mathcal{P}_x(f+f_c)\right], & |f|>f_c \\ 0, & f \text{ elsewhere} \end{Bmatrix}$$



(b.) The normalized average power is

$$P = \int_{-\infty}^{\infty} \mathcal{P}_v(f)\,df \underset{\text{Area under } \mathcal{P}_v(f)}{=} 2\left[\frac{1}{2}(200)(5.0k)\right] = 1{,}000k \text{ Watts}$$

$\underline{P = 1{,}000k \text{ Watts (normalized)}}$

$\boxed{6\text{-}42.}$

$$s(t) = x(t)\cos(\omega_c t + \theta_c) - y(t)\sin(\omega_c t + \theta_c)$$

$$= s_{USSB}(t) + s_{LSSB}(t)$$

where

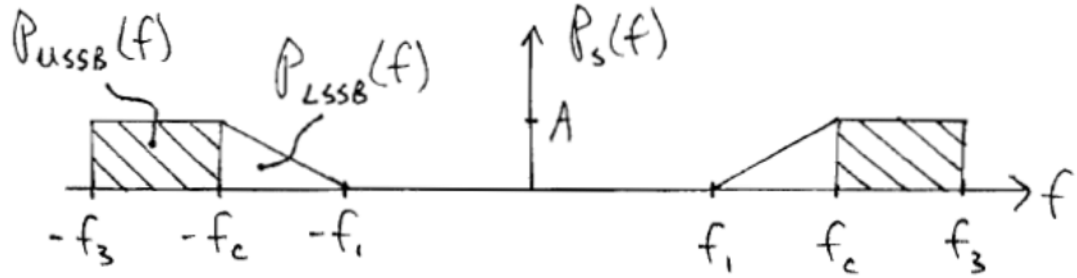
$$s_{USSB}(t) = m_1(t)\cos(\omega_c t + \theta_c) - \hat{m}_1(t)\sin(\omega_c t + \theta_c)$$

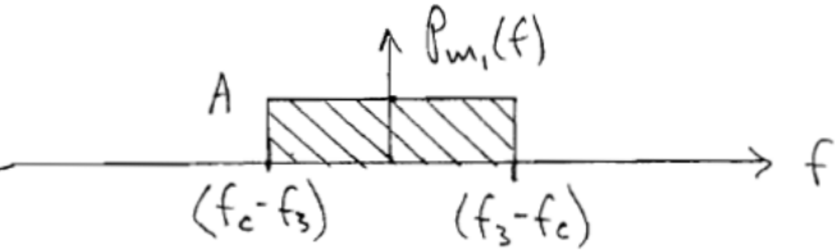
$$s_{LSSB}(t) = m_2(t)\cos(\omega_c t + \theta_c) + \hat{m}_2(t)\sin(\omega_c t + \theta_c)$$

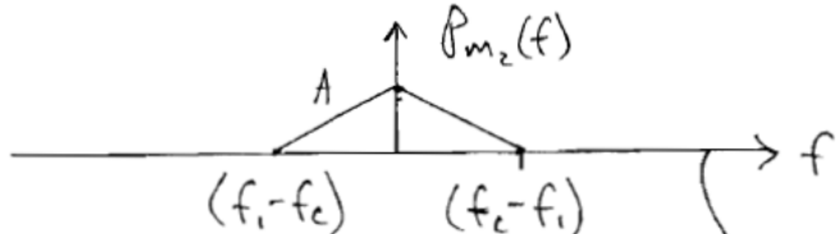
6-42. Cont'd

$$\Rightarrow s(t) = [m_1(t) + m_2(t)]\cos(\omega_c t + \theta_c) - [\hat{m}_1(t) - \hat{m}_2(t)]\sin(\omega_c t + \theta_c)$$

$$\Rightarrow \underline{\underline{x(t) = m_1(t) + m_2(t)}}\ ; \ \underline{\underline{y(t) = \hat{m}_1(t) - \hat{m}_2(t)}}$$







$$\underline{\underline{P_{m_1}(f) = A\,\Pi\left(\frac{f}{2(f_3 - f_c)}\right)}}$$

$$\underline{\underline{P_{m_2}(f) = A\,\Lambda\left(\frac{f}{f_c - f_1}\right)}}$$

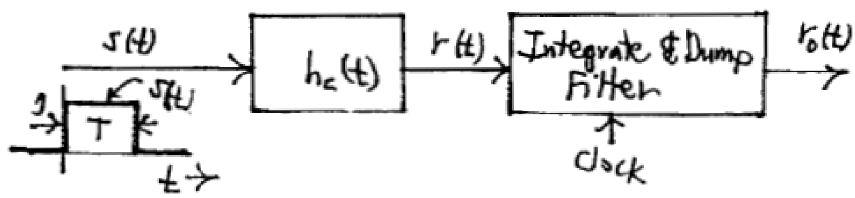
6-44. From example 6-9:

$$P_v(f) = \frac{1}{4}\left[P_x(f-f_c) + P_x(-f-f_c) \right]$$

Where, from solution to problem 6-18.:

$$P_x(f) = T_b \left[\frac{1 - \cos(\pi f T_b)}{\pi f T_b} \right]^2 = P_x(-f)$$

6-48.



$$H_c(f) = \frac{B}{B+jf} = \frac{1}{1 + j\left(\frac{f}{B}\right)} \qquad \text{Let } a = 2\pi B$$

Using Table 2-2

$$\Rightarrow \quad h_c(t) = \begin{cases} 2\pi B e^{-2\pi B t}, & t>0 \\ 0, & t<0 \end{cases} = \begin{cases} a e^{-at}, & t>0 \\ 0, & t<0 \end{cases}$$

$$r(t) = s(t) * h_c(t) = \int_0^t s(\lambda) h_c(t-\lambda)\, d\lambda = \begin{cases} \int_0^t a e^{-a(t-\lambda)} d\lambda, & 0<t<T \\ \int_0^T a e^{-a(t-\lambda)} d\lambda, & t>T \end{cases}$$

$$\Rightarrow \quad r(t) = \begin{cases} 1 - e^{-at}, & 0<t<T \\ e^{-at}\left[e^{aT} - 1\right], & t>T \end{cases}$$

$$r_o(t) = \begin{cases} \int_0^t r(\lambda)\, d\lambda, & 0<t<T \\ \int_T^t r(\lambda)\, d\lambda, & T<t<2T \end{cases} = \begin{cases} \int_0^t \left[1 - e^{a\lambda}\right] d\lambda, & 0<t<T \\ (e^{aT}-1)\int_T^t e^{-a\lambda} d\lambda, & T<t<2T \end{cases}$$

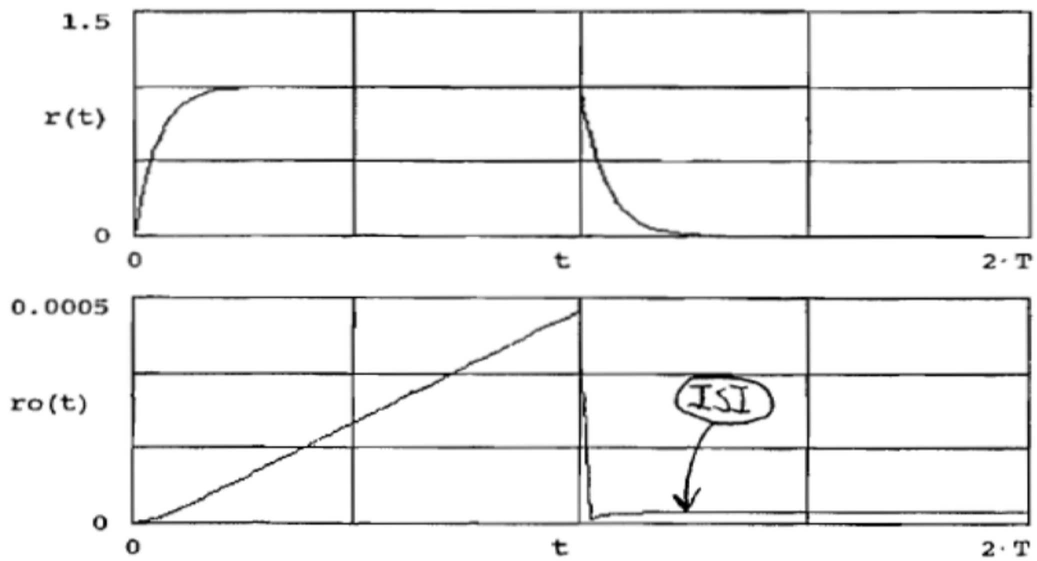
$$\Rightarrow \quad r_o(t) = \begin{cases} t + \frac{1}{a}\left(e^{-at} - 1\right), & 0<t<T \\ \frac{1}{a}\left(e^{aT}-1\right)\left(e^{-aT} - e^{at}\right), & T<t<2T \end{cases}$$

(a.)

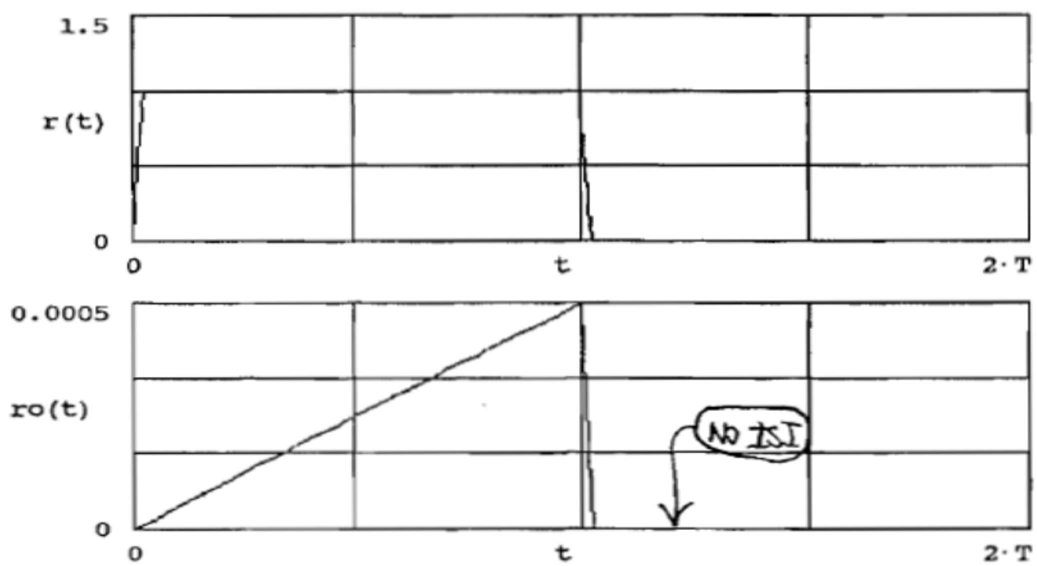
$$R := 2000 \qquad T := \frac{1}{R} \qquad B := 6000 \qquad a := 2\cdot\pi\cdot B \qquad t := 0, \frac{T}{40} \,..\, 2\,T$$

$$r(t) := \text{if}\left[t \le T, 1 - e^{-a\,t}, \left[e^{a\,T} - 1\right]\cdot e^{-a\cdot t}\right]$$

$$ro(t) := \text{if}\left[t \le T, t + \frac{e^{-a\cdot t} - 1}{a}, \frac{e^{a\cdot T} - 1}{a}\cdot\left[e^{-a\,T} - e^{-a\cdot t}\right]\right]$$



(b.) For all pass channel ⇒ Let $B \to \infty$. ⇒ Use $B = 100{,}000$.
Then, results are as follows.

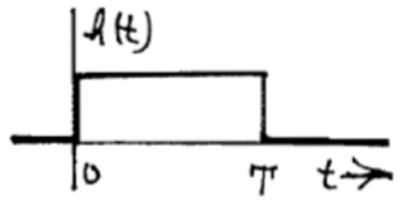


6-50. (a.) $y(t) = \int_{-\infty}^{t} [x(t_1) - x(t_1 - T)]\,dt_1$

For the impulse response let $x(t) = \delta(t)$.

$$\Rightarrow y(t) = h(t) = \int_{-\infty}^{t} [\delta(t_1) - \delta(t_1 - T)]\,dt_1$$

$$h(t) = \begin{cases} \frac{1}{2}, & t = 0 \\ 1, & 0 < t < T \\ \frac{1}{2}, & t = T \\ 0, & t \text{ elsewhere} \end{cases}$$



(b.) $s(t) = h(t_0 - t)$ where $t_0 = T$ $\Rightarrow$ $s(t) = h(t)$ above.

6-51. (a.) $s(t) = s_1(t) = A\cos\omega_1 t$

$$w(t) = s_1(t)\cos(\omega_1 t) = A\cos^2(\omega_1 t) = \frac{1}{2}A[1 + \cos(2\omega_1 t)]$$

$$v_1(t) = \int_0^t w_1(\lambda)\,d\lambda = \frac{1}{2}A\int_0^t [1 + \cos(2\omega_1\lambda)]\,d\lambda = \frac{1}{2}A\left[\lambda + \frac{\sin(2\omega_1\lambda)}{2\omega_1}\right]_0^t$$

$$\Rightarrow \underline{\underline{v_1(t) = \frac{1}{2}A\left[t + \frac{\sin(2\omega_1 t)}{2\omega_1}\right]}}$$

$$w_2(t) = s_1(t)\cos(\omega_2 t) = A\cos(\omega_1 t)\cos(\omega_2 t)$$

$$= \frac{1}{2}A[\cos(\omega_1 - \omega_2)t + \cos(\omega_1 + \omega_2)t]$$

or $w_2(t) = \frac{1}{2}A[\cos(2\Delta\omega t) + \cos((\omega_1 + \omega_2)t)]$ where $\Delta\omega = 2\pi\Delta F$

$$v_2(t) = \int_0^t w_2(\lambda)\,d\lambda = \frac{1}{2}A\int_0^t [\cos(2\Delta\omega\lambda) + \cos(\omega_1 + \omega_2)\lambda]\,d\lambda$$

$$\Rightarrow \underline{\underline{v_2(t) = \frac{1}{2}A\left[\frac{\sin(2\Delta\omega t)}{2\Delta\omega} + \frac{\sin(\omega_1 + \omega_2)t}{\omega_1 + \omega_2}\right]}}$$

$$\underline{\underline{r_0(t) = v_1(t) - v_2(t)}}$$

(b.) $r(t) = s(t) = A\cos\omega_1 t$, $0 < t < T$ $\Rightarrow h(t) = A\cos[\omega_1(T - t)]$

Thus, $r_0(t) = r(t) * h(t) = \int_0^t r(\lambda)h(t - \lambda)$, $0 < t < T$

6-51. (b.) Cont'd.

$$r_0(t) = A^2 \int_0^t \cos(\omega_1\lambda)\cos[\omega_1(T-(t-\lambda))]\,d\lambda$$

$$= \tfrac{1}{2}A^2 \int_0^t [\cos(\omega_1 T - \omega_1 t) + \cos(2\omega_1\lambda + \omega_1 T - \omega_1 t)]\,d\lambda$$

$$= \tfrac{1}{2}A^2 \left[\lambda\cos(\omega_1 T - \omega_1 t) + \frac{\sin(2\omega_1\lambda + \omega_1 T - \omega_1 t)}{2\omega_1}\right]\Bigg|_0^t$$

$$= \tfrac{1}{2}A^2 \left[t\cos(\omega_1 T - \omega_1 t) + \frac{\sin(2\omega_1 t + \omega_1 T - \omega_1 t) - \sin(\omega_1 T - \omega_1 t)}{2\omega_1}\right]$$

Thus,

$$\underline{\underline{r_0(t) = \tfrac{1}{2}A^2\left[t\cos\omega_1(T-t) + \frac{\sin\omega_1(T+t) - \sin\omega_1(T-t)}{2\omega_1}\right]}}$$

$f_c := 1000 \quad \delta f := 50 \quad \omega1 := 2\cdot\pi\cdot\left[f_c - \delta f\right] \quad \omega2 := 2\cdot\pi\cdot\left[f_c + \delta f\right]$

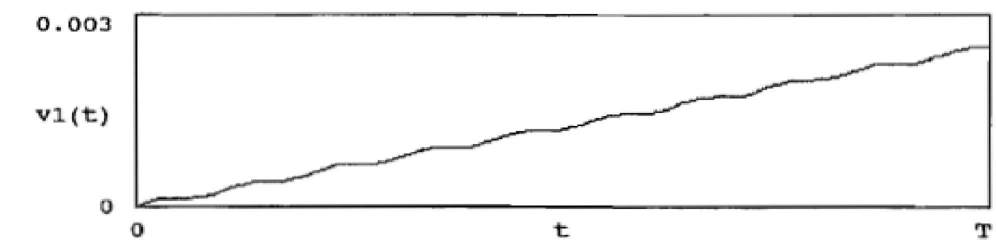
$A := 1 \quad T := \dfrac{1}{4\ \delta f} \quad t := 0, \dfrac{T}{100}\ ..T \quad \delta\omega := 2\cdot\pi\cdot\delta f$

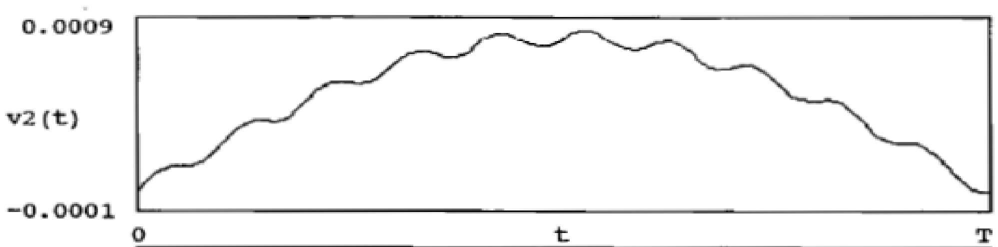
(a.)

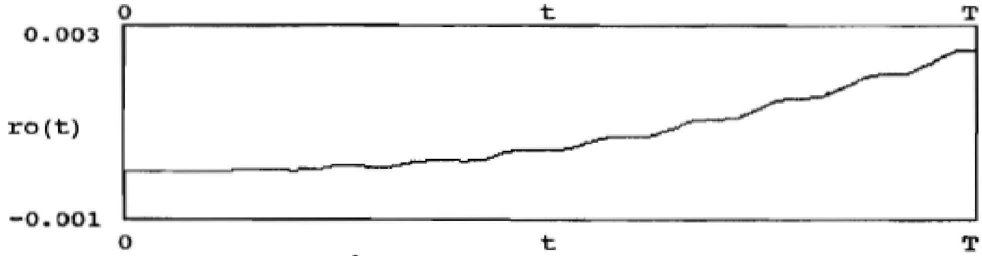
$$v1(t) := \frac{A}{2}\cdot\left[t + \frac{\sin(2\cdot\omega1\cdot t)}{2\cdot\omega1}\right]$$

$$v2(t) := \frac{A}{2}\cdot\left[\frac{\sin((\omega1+\omega2)\cdot t)}{\omega1+\omega2} + \frac{\sin(2\cdot\delta\omega\cdot t)}{2\cdot\delta\omega}\right]$$

$ro(t) := v1(t) - v2(t) \qquad T = 0.005$





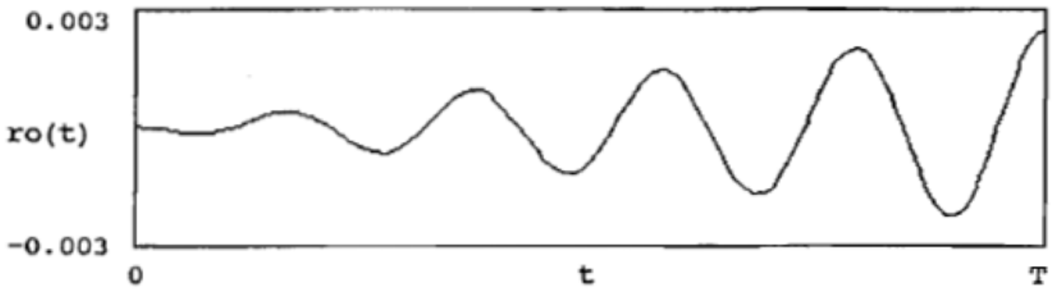


$ro(0.5\ T) = 4.123\cdot 10^{-4}$ $\qquad$ $\underline{\underline{ro(T) = 0.003}}$

6-51. Cont'd.

(b.)

$$ro(t) := \frac{A^2}{2}\cdot\left[t\cdot\cos(\omega 1\ (T - t)) + \frac{\sin(\omega 1\ (T + t)) - \sin(\omega 1\cdot(T - t))}{2\ \omega 1}\right]$$



$ro(0.5\cdot T) = -8.839\cdot 10^{-4}$ $\qquad$ $\underline{\underline{ro(T) = 0.003}}$

(c.) The results for parts (a.) and (b.) are different for $0 < t < T$. However, <u>at the sampling time, $t=T$, the results are identical</u>. That is, Math CAD computes $r_0(T)$ to be 0.003 for both cases.