

Chapter 7

$$\boxed{7-1.} \quad (a.) \quad r_o = \begin{cases} A + n_o, & s_1 \text{ sent} \\ -A + n_o, & s_2 \text{ sent} \end{cases}$$

$$\Rightarrow f(r_o | s_1) = \frac{1}{\sqrt{2} \Delta_o} e^{-\frac{\sqrt{2} |r_o - A|}{\Delta_o}}$$

$$f(r_o | s_2) = \frac{1}{\sqrt{2} \Delta_o} e^{-\frac{\sqrt{2} |r_o + A|}{\Delta_o}}$$

Using (7-8)

$$P_e = \frac{1}{2} \int_{-\infty}^{V_T} \frac{1}{\sqrt{2} \Delta_o} e^{-\frac{\sqrt{2} |r_o - A|}{\Delta_o}} dr_o + \frac{1}{2} \int_{V_T}^{\infty} \frac{1}{\sqrt{2} \Delta_o} e^{-\frac{\sqrt{2} |r_o + A|}{\Delta_o}} dr_o$$

$m_{r_{o1}} = A$, $m_{r_{o2}} = -A$, the source probabilities are equally likely, and the conditional probabilities have symmetrical shapes about $\pm A$.

Thus $\boxed{V_T = 0}$.

$$\therefore P_e = \frac{1}{2\sqrt{2} \Delta_o} \left[\int_{-\infty}^0 e^{-\frac{\sqrt{2} |r_o - A|}{\Delta_o}} dr_o + \int_0^{\infty} e^{-\frac{\sqrt{2} |r_o + A|}{\Delta_o}} dr_o \right]$$

$$= \frac{1}{2\sqrt{2} \Delta_o} \left[\int_{-\infty}^0 e^{\frac{\sqrt{2} (r_o - A)}{\Delta_o}} dr_o + \int_0^{\infty} e^{\frac{-\sqrt{2} (r_o + A)}{\Delta_o}} dr_o \right]$$

$$\text{Let } x_1 = \frac{\sqrt{2} (r_o - A)}{\Delta_o} \text{ and } x_2 = \frac{-\sqrt{2} (r_o + A)}{\Delta_o}$$

$$dx_1 = \frac{\sqrt{2}}{\Delta_o} dr_o$$

$$dx_2 = \frac{-\sqrt{2}}{\Delta_o} dr_o$$

$$= \frac{1}{2\sqrt{2} \Delta_o} \left[\int_{-\infty}^{-\sqrt{2}A/\Delta_o} e^{x_1} \left(\frac{\Delta_o}{\sqrt{2}} dx_1 \right) + \int_{-\sqrt{2}A/\Delta_o}^{-\infty} e^{x_2} \left(\frac{-\Delta_o}{\sqrt{2}} dx_2 \right) \right]$$

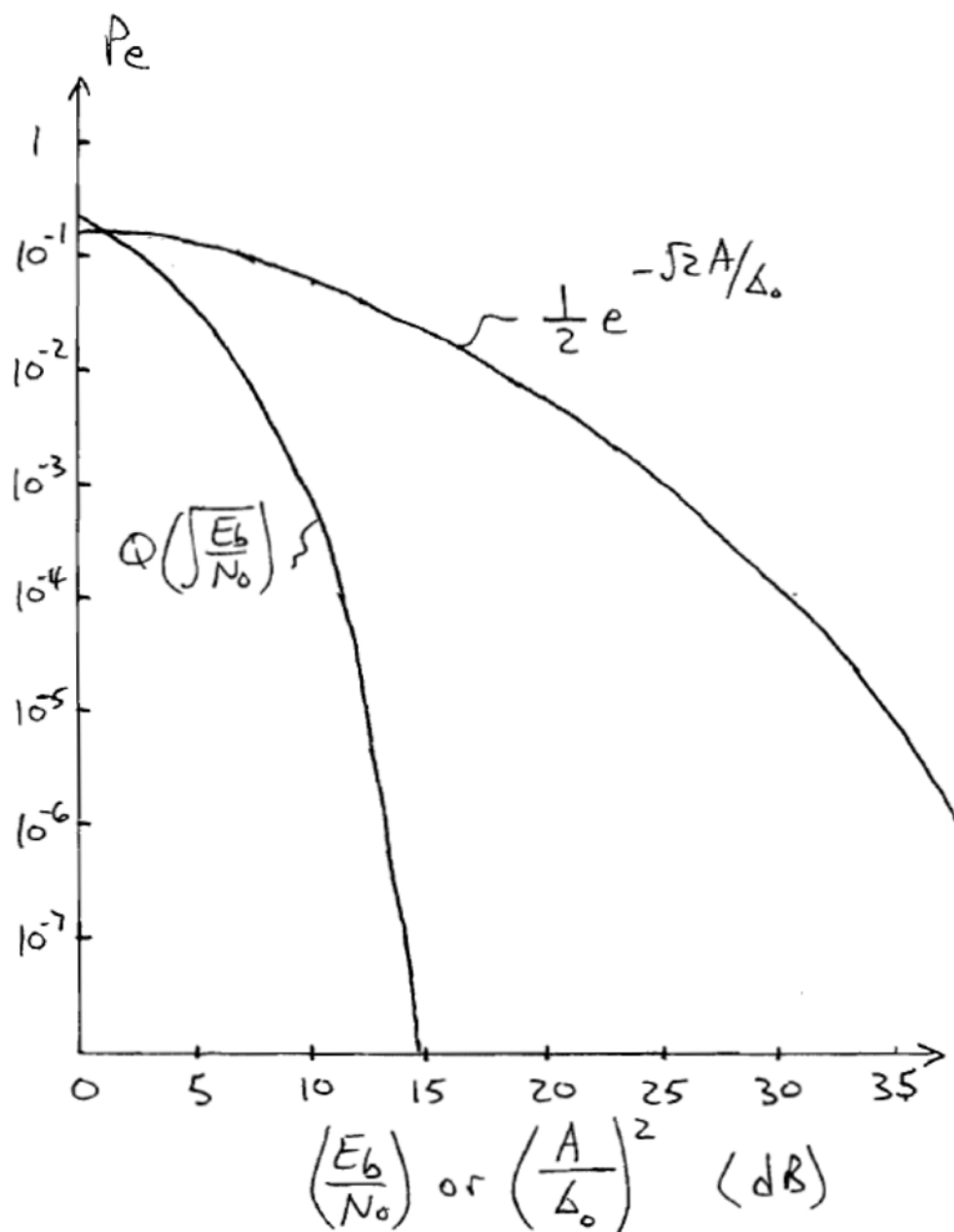
$$= \frac{1}{4} \left[\int_{-\infty}^{-\sqrt{2}A/\Delta_o} e^{x_1} dx_1 + \int_{-\infty}^{-\sqrt{2}A/\Delta_o} e^{x_2} dx_2 \right]$$

7-1. (a.) Cont'd

$$P_e = \frac{1}{2} \left[\int_{-\infty}^{-\sqrt{2}A/\Delta_0} e^x dx \right] = \frac{1}{2} e^x \Big|_{-\infty}^{-\sqrt{2}A/\Delta_0}$$

$$= \frac{1}{2} \left[e^{-\sqrt{2}A/\Delta_0} - e^{-\infty} \right] = \underline{\underline{\frac{1}{2} e^{-\sqrt{2}A/\Delta_0}}} = P_e$$

(b.)



P_e much larger for Laplacian Noise.

$$\boxed{7-5.} \quad \left(\frac{S}{N}\right)_{in} = \frac{\frac{E_b}{T_b}}{\left(\frac{N_0}{2}\right)(2B_{eq})} = \frac{E_b R}{N_0 B_{eq}} \Rightarrow \frac{E_b}{N_0} = \frac{B_{eq}}{R} \left(\frac{S}{N}\right)_{in}$$

Aside:

$$B_{eq} = \frac{\int_{-\infty}^{\infty} |H(f)|^2 df}{2 |H(0)|^2} = \frac{\left(\frac{2K}{N_0}\right)^2 \int_{-\infty}^{\infty} |S'(f)|^2 df}{2 \left(\frac{2K}{N_0}\right)^2 |S'(0)|^2} = \frac{\int_{-\infty}^{\infty} |S'(f)|^2 df}{2 |S'(0)|^2}$$

Using (6-155) for MF: $H(f) = \frac{K S'(f) e^{-j\omega t_0}}{N_0/2}$

$$\Rightarrow B_{eq} = \frac{T_b \int_{-\infty}^{\infty} \left(\frac{\sin(\pi T_b f)}{\pi T_b f}\right)^2 df}{2 T_b^2} = \frac{1}{2\pi T_b} \int_{-\infty}^{\infty} \left(\frac{\sin x}{x}\right)^2 dx = \frac{1}{2\pi T_b} \pi = \frac{1}{2} R$$

$s(t) = \pi\left(\frac{t}{T_b}\right) \leftrightarrow S'(f) = T_b \frac{\sin(\pi T_b f)}{\pi T_b f}$

Let $x = \pi T_b f$
 $dx = \pi T_b df$

$$\Rightarrow \frac{E_b}{N_0} = \frac{1}{2} \frac{R}{R} \left(\frac{S}{N}\right)_{in} = \frac{1}{2} \left(\frac{S}{N}\right)_{in} = \frac{E_b}{N_0}$$

Using (7-24b):

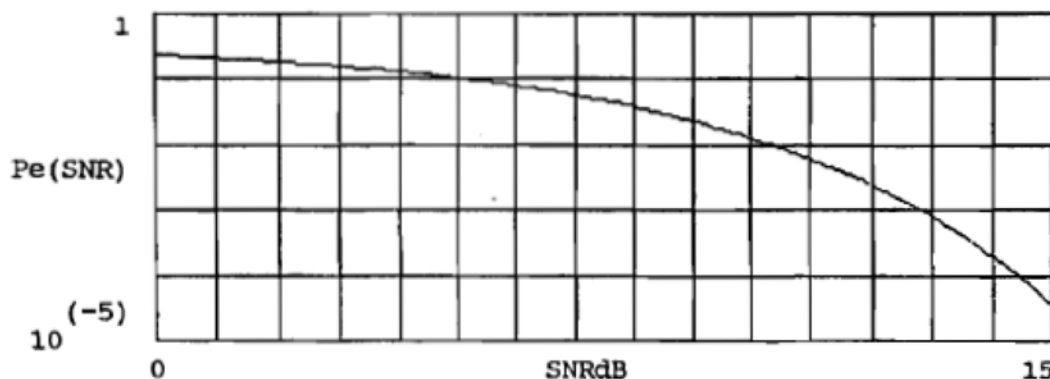
$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{1}{2} \left(\frac{S}{N}\right)_{in}}\right)$$

SNRdB := 0, 0.1 .. 15

Q(x) := 1 - cnorm(x)

SNR(SNRdB) := 10 ^{$\frac{\text{SNRdB}}{10}$}

Pe(SNR) := Q[$\sqrt{0.5 \cdot \text{SNR}(\text{SNRdB})}$]



7-8. (a.) For derivation of P_e , follow the same procedure as used in the solution for Prob. 7-7.

$$B_{eq} = \int_0^\infty \frac{|H(f)|^2}{|H(0)|^2} df = \int_0^\infty \frac{1}{1 + (\frac{f}{f_0})^4} df = f_0 \int_0^\infty \frac{1}{1 + x^4} dx = \frac{f_0 \pi}{2\sqrt{2}}$$

Let $x = f/f_0$
Using Sec. A-3

$$S_{01} = \int_0^T S_{01}(T-\lambda) h(\lambda) d\lambda = \int_0^T A [\sqrt{2} \omega_0 e^{-(\omega_0 \sqrt{2})\lambda} \sin(\frac{\omega_0}{\sqrt{2}}\lambda)] d\lambda$$

or $S_{01} = \int_0^T 2A e^{-x} \sin(x) dx = A [1 - e^{-\sqrt{2}\pi} (\sin(\sqrt{2}\pi) + \cos(\sqrt{2}\pi))]$

Let $x = \frac{\omega_0}{\sqrt{2}}\lambda$
Using Sec. A-5

$$\Rightarrow S_{01} = 1.01447 A$$

$$f_0 = \frac{1}{T}$$

$$\sigma_0^2 = \frac{N_0}{2} B_{eq} = N_0 B_{eq} = \frac{N_0 \pi f_0}{2\sqrt{2}} = \frac{N_0 \pi}{2\sqrt{2} T}$$

Using (7-17) where $S_{01} = -S_{02}$,

$$P_e = Q\left(\sqrt{\frac{S_{01}^2}{\sigma_0^2}}\right) = Q\left(\sqrt{\frac{(1.01447)^2 A^2}{\frac{N_0 \pi}{2\sqrt{2} T}}}\right) = Q\left(\sqrt{\frac{2\sqrt{2} (1.01447)^2 A^2 T}{\pi N_0}}\right)$$

$$\Rightarrow P_e = Q\left(\sqrt{\frac{2\sqrt{2} (1.01447)^2}{\pi} \left(\frac{E_b}{N_0}\right)}\right) = \underline{\underline{Q\left(\sqrt{0.92656 \left(\frac{E_b}{N_0}\right)}\right)}}$$

(b.) EbNodB := 0, 0.1 .. 15

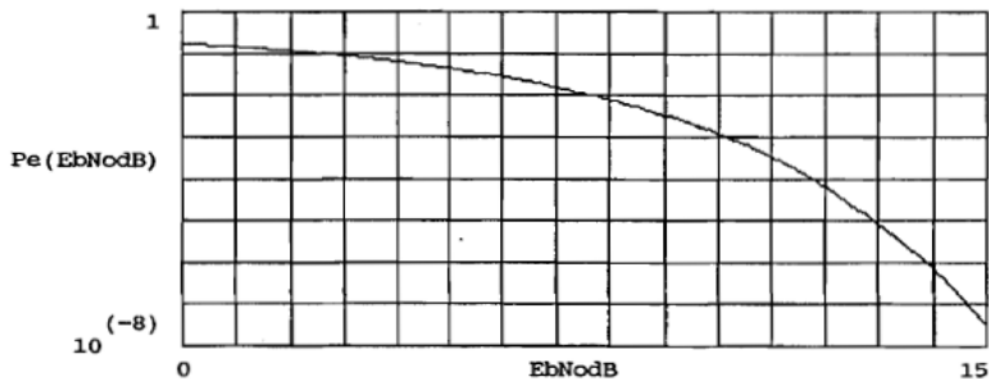
Q(x) := 1 - cnorm(x)

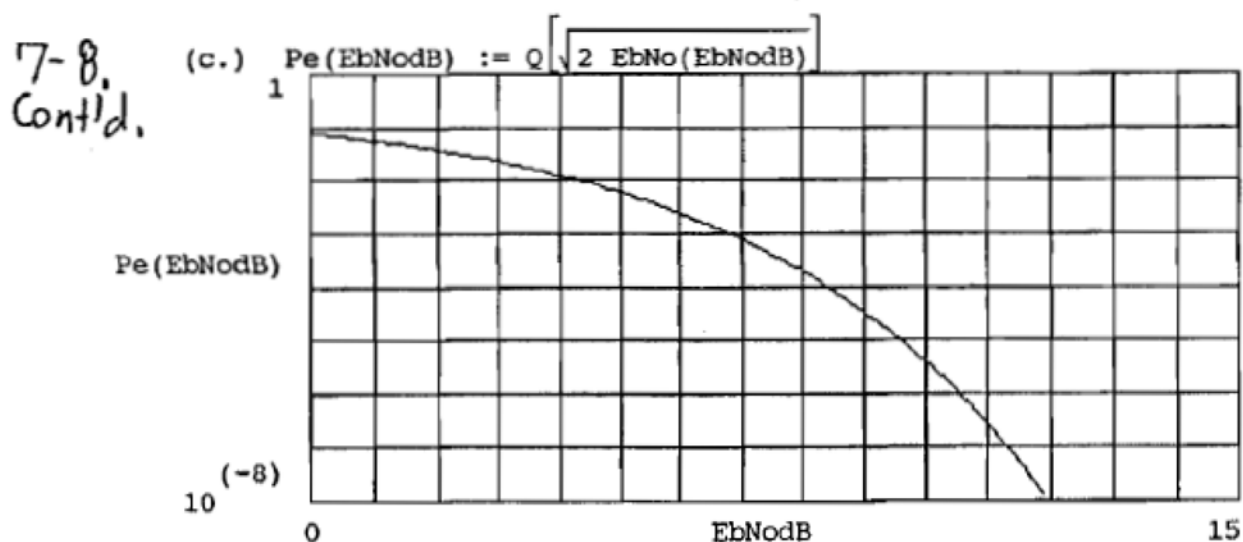
EbNodB

10

EbNo(EbNodB) := 10

Pe(EbNodB) := Q[$\sqrt{0.92656 \cdot \text{EbNo}(\text{EbNodB})}$]





7-11.

(a.) Referring to the solution for SA 7-3,

$$P_e = Q\left(\sqrt{\frac{A^2}{4N_bB}}\right) \quad (7-24a)$$

where $B = \frac{2}{T} = 2R$ and $E_b = \left(\frac{A^2}{2}\right)/\pi = \frac{A^2}{2R}$

Thus,

$$P_e = Q\left(\sqrt{\frac{A^2}{8N_bR}}\right) = Q\left(\sqrt{\frac{A^2}{4N_b2R}}\right) = Q\left(\sqrt{\frac{1}{4}\left(\frac{E_b}{N_b}\right)}\right)$$

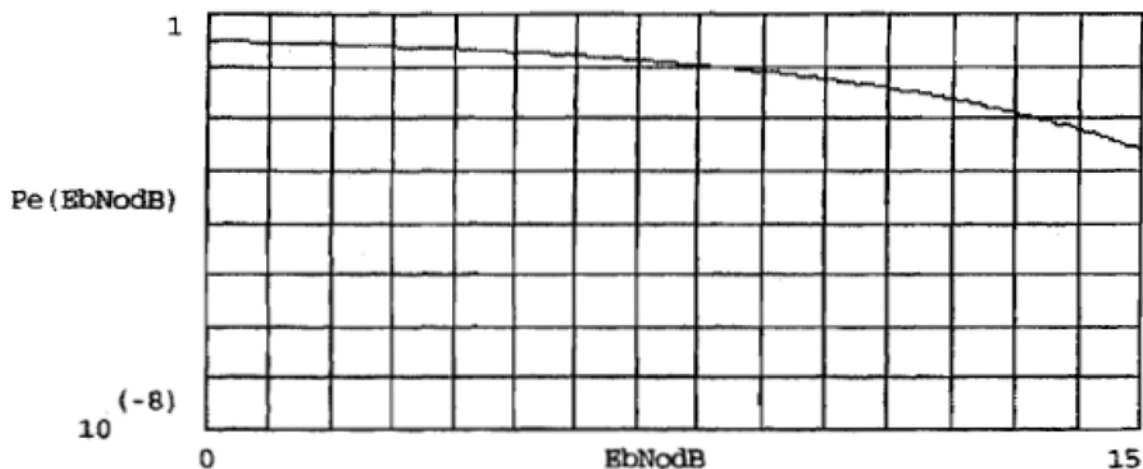
$E_b N_{dB} := 0, 0.1 \dots 15$

$Q(x) := 1 - \text{cnorm}(x)$

$E_b N_{dB}$
 $\frac{E_b N_{dB}}{10}$
 $E_b N_{dB} := 10$

(a.)

$P_e(E_b N_{dB}) := Q\left[\sqrt{0.25 \cdot E_b N_{dB}}\right]$ <---- LPF with ISI Result



(b.)

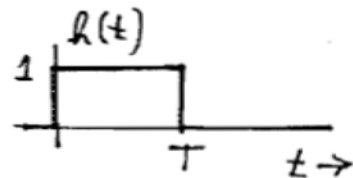
$P_e(E_b N_{dB}) := Q\left[\sqrt{E_b N_{dB}}\right]$ Matched Filter Result <-- (7-24b)



7-14.

(a) The impulse response is:

$$h(t) = s_{01}(T-t) = \underline{s_{01}(t)}$$



$$H(f) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt = \int_0^T e^{-j\omega t} dt = \frac{e^{-j\omega t}}{-j\omega} \Big|_0^T = \frac{e^{-j\omega T} - e^{-j\omega \cdot 0}}{-j\omega}$$

$$\Rightarrow H(f) = e^{-j\omega T/2} \left[\frac{e^{j\omega T/2} - e^{-j\omega T/2}}{j\omega} \right] = \underline{T e^{-j\pi f T} \left[\frac{\sin(\pi f T)}{\pi f T} \right]}$$

See Fig. 6-17

$$(b) \quad B_{eq} = \frac{\int_0^{\infty} |H(f)|^2 df}{|H(0)|^2} = \frac{T^2 \int_0^{\infty} \left[\frac{\sin(\pi f T)}{\pi f T} \right]^2 df}{T^2}$$

$$= \int_0^{\infty} \left(\frac{\sin x}{x} \right)^2 \left(\frac{1}{\pi T} dx \right) = \frac{1}{\pi T} \left(\frac{\pi}{2} \right) = \underline{\underline{\frac{1}{2T} = B_{eq}}}$$

Let $x = \pi f T$; $dx = \pi T df$

Using Sec. A-5

7-16.

From (7-8)

$$P_e = P(1) \int_{-\infty}^{v_T} f(r_0 | s_1) dr_0 + P(0) \int_{v_T}^{\infty} f(r_0 | s_2) dr_0$$

$$\text{where } r_0 = \begin{cases} A + n_0, & \text{for a binary 1 sent} \\ -A + n_0, & \text{" " " 0 " "} \end{cases}$$

Thus

$$P_e = P(1) \int_{-\infty}^{v_T} \frac{1}{\sqrt{2\pi}\Delta} e^{-\frac{(r_0 - A)^2}{2\Delta^2}} dr_0 + P(0) \int_{v_T}^{\infty} \frac{1}{\sqrt{2\pi}\Delta} e^{-\frac{(r_0 + A)^2}{2\Delta^2}} dr_0$$

$$\text{Let } \lambda_1 = -(r_0 - A)/\Delta \quad ; \quad \lambda_2 = (r_0 + A)/\Delta$$

$$d\lambda_1 = -\frac{1}{\Delta} dr_0 \quad d\lambda_2 = \frac{1}{\Delta} dr_0$$

$$7-16. \text{Cont'd } \Rightarrow P_e = P(1) \int_{(-V_T+A)/\Delta}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\lambda_1^2/2} d\lambda_1 + P(0) \int_{(V_T+A)/\Delta}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\lambda_2^2/2} d\lambda_2$$

$$\text{or } P_e = P(1) Q\left[\frac{-V_T+A}{\Delta}\right] + P(0) Q\left[\frac{V_T+A}{\Delta}\right]$$

$$= P(1) Q\left[\sqrt{\frac{(-V_T+A)^2 2T}{N_0}}\right] + P(0) Q\left[\sqrt{\frac{(V_T+A)^2 2T}{N_0}}\right]$$

$$\Delta^2 = \frac{N_0}{2T}$$

To check: Let $P(1) = P(0) = \frac{1}{2}$; $V_T = 0$

$$P_e = \frac{1}{2} Q\left[\sqrt{\frac{A^2 2T}{N_0}}\right] + \frac{1}{2} Q\left[\sqrt{\frac{A^2 2T}{N_0}}\right]$$

$$= Q\left[\sqrt{\frac{2A^2 T}{N_0}}\right]; \text{ This checks with equ. (7-26b)}$$

7-20. Referring to Fig. 7-7

(a.) Let $s_1(t) = A \cos \omega_c t$, $s_2(t) = -A \cos \omega_c t$

$$n(t) = x(t) \cos \omega_c t - y(t) \sin \omega_c t$$

$$\text{Coherent reference} = 2 \cos(\omega_c t + \theta_e)$$

$$\Rightarrow r_0(t) = \frac{1}{2} A \cos \theta_e + n_0(t) = s_{01}(t) + n_0(t)$$

$$\text{(A-11) where } n_0(t) = x(t) \cos \theta_e + y(t) \sin \theta_e$$

$$\overline{n_0^2(t)} = \overline{x^2(t)} \cos^2 \theta_e + 2 \overline{x(t)y(t)} \cos \theta_e \sin \theta_e + \overline{y^2(t)} \sin^2 \theta_e$$

$$\Rightarrow \overline{n_0^2(t)} = 2 N_0 B (\cos^2 \theta_e + \sin^2 \theta_e) = 2 N_0 B$$

$$\uparrow \overline{x^2} = \overline{y^2} = 2 N_0 B$$

Corresponds to (7-36)

Using (7-17): $H(f) = \text{LPF}$

$$P_e = Q\left[\pm \sqrt{\frac{(s_{01} s_{02})^2}{4 \sigma_0^2}}\right] = Q\left[\pm \sqrt{\frac{4 A^2 \cos^2 \theta_e}{8 N_0 B}}\right] = Q\left[\pm \sqrt{\frac{A^2 \cos^2 \theta_e}{2 N_0 B}}\right]$$

where $\begin{cases} + \text{ is used when } s_{01} > s_{02} \Rightarrow |\theta_e| < \pi/2 \\ - \text{ is used when } s_{02} > s_{01} \Rightarrow \pi/2 < |\theta_e| < \pi \end{cases}$

(b.) If $H(f)$ is matched to the output of the multiplier, then

$$P_e = Q\left(\sqrt{\frac{E_d}{2N_0'}}\right) \quad \text{Using (7-20)}$$

Where $N_0' = 2N_0$ and

$$E_d = \int_0^T [s_{01}(t) - s_{02}(t)]^2 dt = \int_0^T 4A^2 \cos^2 \theta_e dt$$

or $E_d = 4A^2 T \cos^2 \theta_e = 8 E_b \cos^2 \theta_e$

$$\Rightarrow P_e = Q\left[\sqrt{\frac{2(E_b)}{N_0}} \cos^2 \theta_e\right]$$

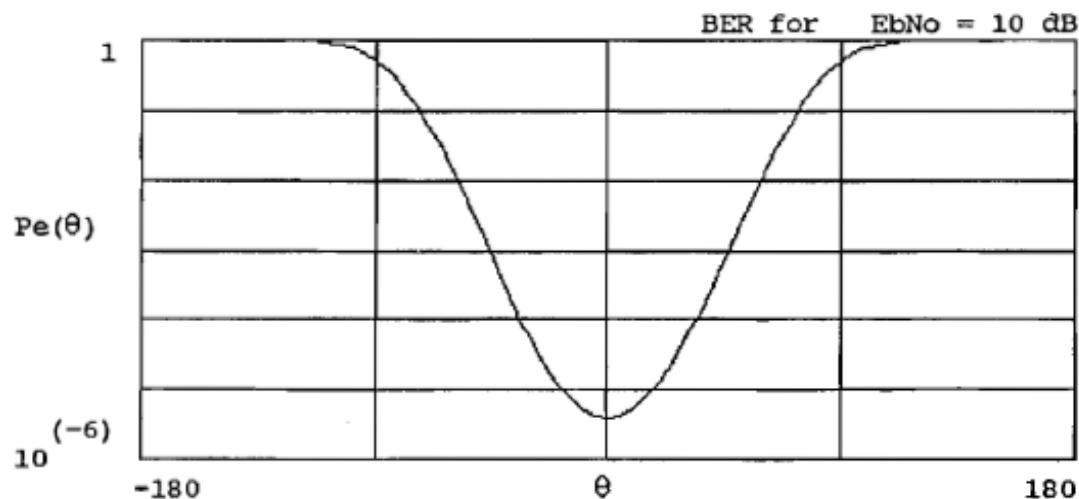
$$E_b = \frac{1}{2} E_{s1} + \frac{1}{2} E_{s2} = \frac{1}{2} \left(\frac{1}{2} A^2 T\right) + \frac{1}{2} \left(\frac{1}{2} A^2 T\right) = \frac{1}{2} A^2 T$$

← Corresponds to (7-38)

(c.) $E_b \text{NodB} := 10 \quad \theta := -180, -175 \dots 180 \quad E_b \text{NodB} = 10$
 $Q(x) := 1 - \text{cnorm}(x) \quad E_b \text{No} := 10$

$$z(\theta) := \sqrt{2 E_b \text{No} \cdot \left[\cos \left[\pi \frac{\theta}{180} \right] \right]^2}$$

$$P_e(\theta) := \text{if}(|\theta| \leq 90, Q(z(\theta)), Q(-z(\theta)))$$



7-21. (a.) Overall $P_e = 10 (P_e)_i = \underline{\underline{5 \times 10^{-7}}}$

(b.) when repeaters were used, the E_b/N_0 at the input to each was described by:

$$P_e = Q \left[\sqrt{2 \left(\frac{E_b}{N_0} \right)} \right] = 5 \times 10^{-8} \xrightarrow{\text{Sec. A-10}} \frac{1}{\sqrt{2\pi \left(\frac{2E_b}{N_0} \right)}} e^{-E_b/N_0} \Rightarrow \frac{E_b}{N_0} = \underline{\underline{14.2}}$$

Now with 10 amplifiers, the Rx input consists of the BPSK signal with E_b energy/bit plus a noise level 10 times that present before (since the line from one amp to the next contributes a PSD of $N_0/2$, and there are 10 such lines). Thus $\left(\frac{E_b}{N_0} \right)' = \frac{14.2}{10} = \underline{\underline{1.42}}$

$$\therefore P_e' = Q \left(\sqrt{2(1.42)} \right) = Q(1.69) \approx Q(1.7) = \underline{\underline{4.4 \times 10^{-2}}}$$

7-22. (a.) $B_T = 2700 - 300 = 2400 \text{ Hz}$ Bandpass System

The largest bit rate that can be accommodated w/o I.S.I. is (Table 7-1.)

$$\underline{\underline{B = R = 2400 \text{ bits/sec}}}$$

(b.) From Table 7.1 for BPSK:

$$P_e = Q \left(\sqrt{2 \left(\frac{E_b}{N_0} \right)} \right) \xrightarrow{\text{Sec. A-10}} \frac{1}{\sqrt{4\pi \left(\frac{E_b}{N_0} \right)}} e^{-E_b/N_0}, \text{ for each repeater}$$

$$\text{But } \left(\frac{S}{N} \right) = \frac{P_s}{N_0 B_T} \xrightarrow{B_T = R} \frac{P_s}{N_0 R} \xrightarrow{R = \frac{1}{T}} \frac{P_s T}{N_0} = \frac{E_b}{N_0} = \frac{S}{N} = 15 \text{ dB}$$

$$\Rightarrow \frac{E_b}{N_0} = 15 \text{ dB} = 31.6$$

$$\therefore P_e = \frac{1}{\sqrt{4\pi(31.6)}} e^{-31.6} = 9.26 \times 10^{-16} / \text{repeater}$$

There are $n = \frac{600 \text{ mi.}}{50 \text{ mi/rep}} = 12 \text{ repeaters (including } R_x)$

$$\therefore \text{Overall } (P_e) \approx n P_e = 12 (9.26 \times 10^{-16}) = \underline{\underline{1.11 \times 10^{-14}}}$$

Note: If there are n repeaters, there is an error at the end of the line only if there are an odd number of errors along the line (for the bit in question).

$$\Rightarrow P(\text{errors}) = \binom{n}{k} P_e^k (1-P_e)^{n-k}; \quad \binom{n}{k} = \frac{n!}{(n-k)! k!}$$

$$\Rightarrow \text{Overall } P_e = \sum_{\substack{k=1 \\ k \text{ odd}}}^n P(\text{errors}) \approx n P_e \quad \text{If } n P_e \ll 1$$

7-27. From Table 7-1 for FSK w/ non coherent detection:

$$P_e = \frac{1}{2} e^{-\frac{1}{2} (E_b / N_{\text{total}})}$$

$$N_{\text{total}} = K (T_0 + T_{\text{eff}}) \approx K (T_0 + (F-1) T_0) = K F T_0$$

$$T_{\text{eff}} = (F-1) T_0 = (1.38 \times 10^{-23}) (10^{6/10}) 290$$

$$\frac{E_b}{N_{\text{total}}} = \frac{P_s T}{N_{\text{tot}} R} \approx \frac{P_s}{N_{\text{tot}} R} = \frac{V_s^2 / R_A}{K F T_0 R} = 28.53$$

$R = 1/T$

$$P_e = \frac{1}{2} e^{-\frac{1}{2} (28.53)} = \underline{\underline{3.2 \times 10^{-7}}}$$

7-31. See Table 7-1.

(a.) For QPSK the largest R and min P_e are
 $R = 2B = 2(2700 - 300) = \underline{\underline{4800 \text{ b/s}}}$ obtained.

$$\frac{S}{N} = \frac{P_s}{\frac{N_0}{2}(2B)} \overset{\substack{\uparrow \\ B = \frac{1}{2}R}}{=} \frac{2P_s}{N_0 R} \overset{\substack{\uparrow \\ R = \frac{1}{T}}}{=} \frac{2P_s T}{N_0} = \frac{2E_b}{N_0}$$

$$\Rightarrow \frac{E_b}{N_0} = \frac{1}{2} \left(\frac{S}{N} \right) = \frac{1}{2} (10^{2.5}) = 158.1 \Rightarrow 22 \text{ dB}$$

$$P_e = Q\left(\sqrt{2\left(\frac{E_b}{N_0}\right)}\right) \overset{\substack{\uparrow \\ \text{Figure 7-14.}}}{\ll} 10^{-5} \text{ for } \frac{E_b}{N_0} = 22 \text{ dB}$$

$$C = B \log_2 \left(1 + \frac{S}{N} \right) = 2400 \log_2 (1 + 10^{2.5})$$

$$= \frac{2400}{\ln 2} \ln(1 + 316.23) = 1.99 \times 10^4 = \underline{\underline{19,900 \text{ b/s}}}$$

7-32. (a.) $R = \left(8 \text{ K } \frac{\text{samples}}{\text{sec}}\right) \left(8 \frac{\text{bits}}{\text{sample}}\right) = 64 \text{ K b/s}$

$$\frac{S}{N} = \frac{P_s}{N_0 B} \uparrow \frac{P_s T_b}{N_0} = \frac{E_b}{N_0} = 10^{.8} = 6.3$$

$\beta = R = \frac{1}{T_b}$ ← Table 7-1. for DPSK :

$$P_e = \frac{1}{2} e^{-\left(\frac{E_b}{N_0}\right)} = \frac{1}{2} e^{-6.3} = \underline{\underline{9.18 \times 10^{-4}}}$$

(b.) Using (7-70) with $m = 2^8 = 256$:

$$\begin{aligned} \left(\frac{S}{N}\right)_{\text{out}} &= \frac{3m^2}{1 + 4(m^2 - 1)P_e} = \frac{3(256)^2}{1 + 4[(256)^2 - 1]9.18 \times 10^{-4}} \\ &= 813.6 \Rightarrow \underline{\underline{29.1 \text{ dB}}} \end{aligned}$$

7-34. Using (7-89),

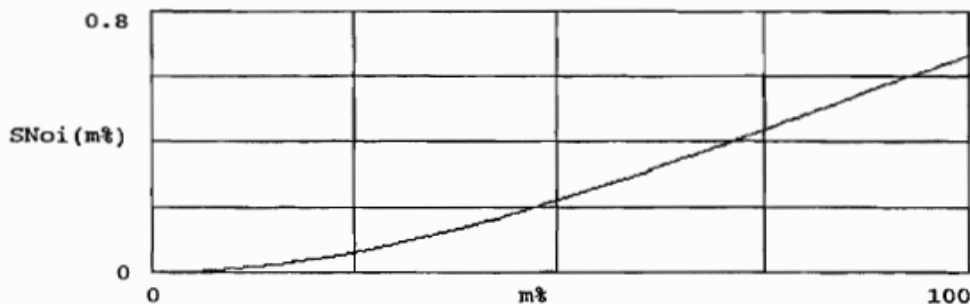
$$S_{N_{oi}} \triangleq \frac{(S/N)_{\text{out}}}{(S/N)_{\text{in}}} = \frac{2\overline{m^2}}{1 + \overline{m^2}}$$

Let $m(t) = \left(\frac{m\%}{100}\right) \cos \omega_m t \Rightarrow \overline{m^2} = \frac{1}{2} \left(\frac{m\%}{100}\right)^2$

$$\Rightarrow S_{N_{oi}} = \frac{\left(\frac{m\%}{100}\right)^2}{1 + \frac{1}{2} \left(\frac{m\%}{100}\right)^2}$$

$m\% := 0.5 \dots 100$

$$S_{N_{oi}}(m\%) := \frac{\left[\frac{m\%}{100}\right]^2}{1 + 0.5 \left[\frac{m\%}{100}\right]^2}$$



7-35. $m(t) = 0.4 \sin \omega_m t \Rightarrow \overline{m^2} = \frac{(0.4)^2}{2} = 0.08$

For AM, with product detector, use (7-90).

$$\frac{(S/N)_{\text{out}}}{(S/N)_{\text{base}}} = \frac{\overline{m^2}}{1 + \overline{m^2}} = \frac{0.08}{1.08} = 0.0741 \Rightarrow \underline{\underline{-11.3 \text{ dB}}}$$

Also get same result for env. det when (S/N) is large.

For DSB-SC, use (7-98).

$$\frac{(S/N)_{\text{out}}}{(S/N)_{\text{base}}} = 1 \Rightarrow \underline{\underline{0 \text{ dB}}} \Rightarrow \underline{\underline{\text{The AM system is inferior by 11.3 dB}}}$$

7-42. Using Carson's Rule:

$$B_{\text{IF}} = 2(\beta_f + 1)B \Rightarrow 25 \text{ KHz} = 2(\beta_f + 1)5 \text{ KHz} \Rightarrow \beta_f = 1.5$$

$$f_1 = 2.1 \text{ KHz} ; \frac{B}{f_1} = \frac{5}{2.1} \gg 1 \therefore (7-139) \text{ is not valid}$$

Eqn. (7-124a) $s_o(t) = \frac{K D_f}{2\pi} m(t) = \frac{K B \beta_f}{V_p} m(t)$

$$\overline{s_o^2(t)} = K^2 B^2 \beta_f^2 \overline{\left(\frac{m}{V_p}\right)^2} ; \overline{\left(\frac{m}{V_p}\right)^2} = \frac{1}{2} \text{ for sinusoid}$$

Eqn. (7-136) $\overline{[\tilde{n}_o(t)]^2} = 2 \left(\frac{K}{A_c}\right)^2 N_o f_1^3 \left[\frac{B}{f_1} - \tan^{-1}\left(\frac{B}{f_1}\right) \right]$

$$\therefore \left(\frac{S}{N}\right)_o = \frac{K^2 B^2 \beta_f^2 \overline{\left(\frac{m}{V_p}\right)^2}}{2 \left(\frac{K}{A_c}\right)^2 N_o f_1^3 \left[\frac{B}{f_1} - \tan^{-1}\left(\frac{B}{f_1}\right) \right]}$$

$$\text{Eqn. (7-128)} \quad \left(\frac{S}{N}\right)_{in} = \frac{A_c^2}{4N_o(\beta_f+1)B}$$

$$\frac{(S/N)_o}{(S/N)_{in}} = \frac{2\left(\frac{B}{f_i}\right)^3 \beta_f^2 (\beta_f+1) \left(\frac{m}{J_P}\right)^2}{\left[\frac{B}{f_i} - \tan^{-1}\left(\frac{B}{f_i}\right)\right]} = \frac{2(13.5)(2.25)(2.5)\frac{1}{2}}{1.21} = 62.9$$

$$N_{total} = kFT_o ; F = 10^{1.2} = 15.8$$

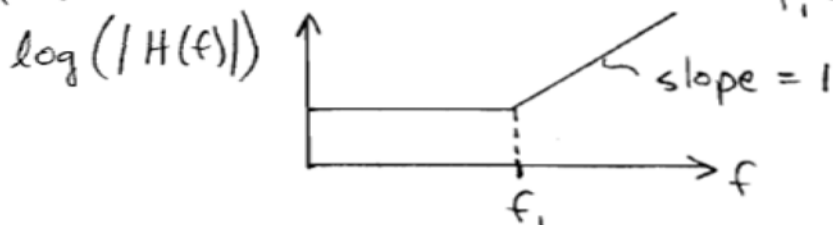
$$\left(\frac{S}{N}\right)_{in} = \frac{P_s}{\frac{N_{tot}}{2}(2B_{IF})} = \frac{P_s}{kFT_o B_{IF}} = \frac{P_s}{1.57 \times 10^{-15}}$$

$$\Rightarrow P_s = \left(\frac{S}{N}\right)_o \left(\frac{1}{62.9}\right) (1.57 \times 10^{-15})$$

$$= 10^{3.5} \left(\frac{1}{62.9}\right) (1.57 \times 10^{-15}) = 7.89 \times 10^{-14} \text{ W}$$

$$10 \log_{10} \left(\frac{7.89 \times 10^{-14}}{10^{-3}} \right) = \underline{\underline{-101 \text{ dBm}}} = P_{s \text{ min}}$$

7-47. $H(f) = [1 + j f/f_i]$ for preemphasis
(a.) $f_i = 2.1 \text{ kHz}$



At $f = 15 \text{ kHz}$ the gain is:

$$|H(f)| = \left| 1 + j \left(\frac{15}{2.1} \right) \right| = \sqrt{1 + \left(\frac{15}{2.1} \right)^2} = 7.21$$

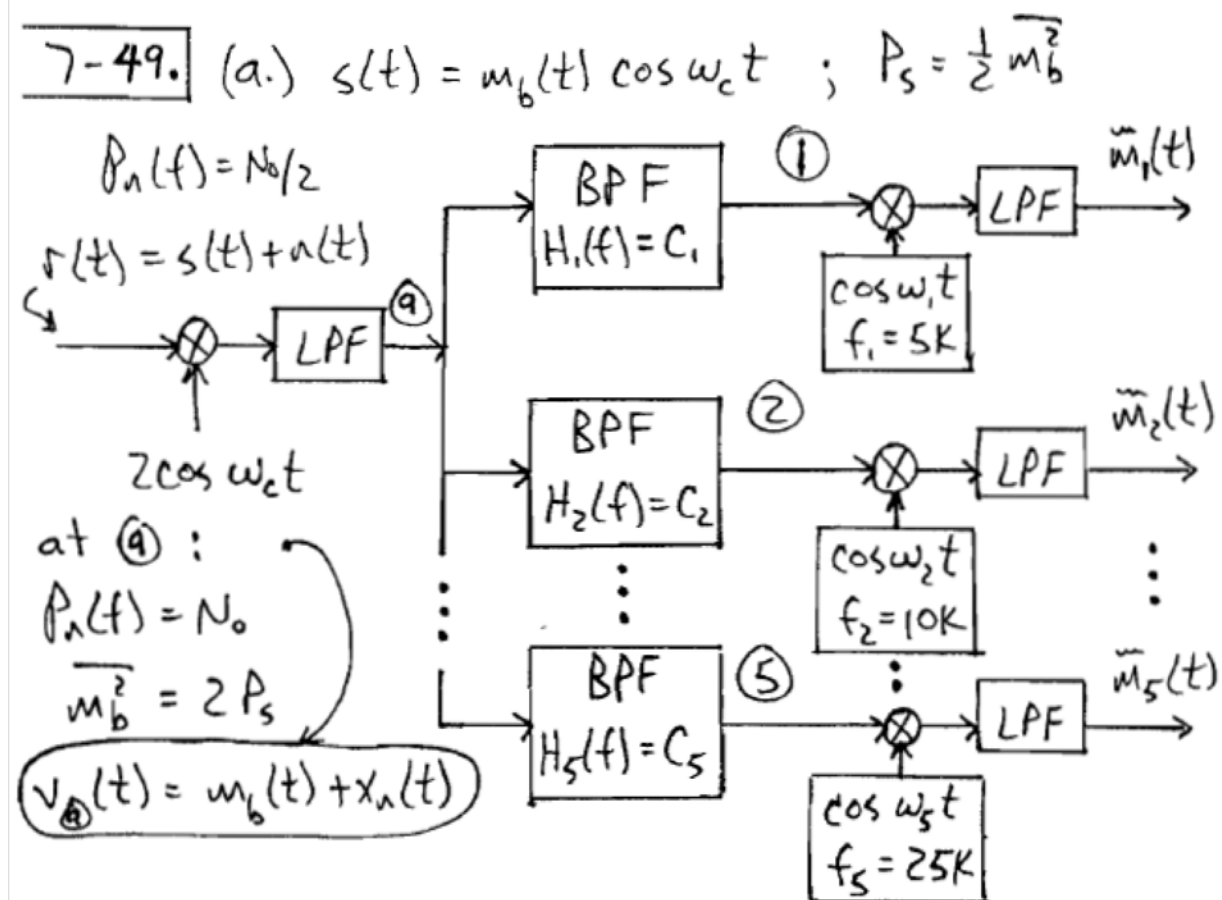
At $f = 1 \text{ kHz}$:

$$|H(f)| = \left| 1 + j \left(\frac{1}{2.1} \right) \right| = \sqrt{1 + \left(\frac{1}{2.1} \right)^2} = 1.10$$

$$\Delta F = 75 \text{ kHz} \left(\frac{7.21}{1.10} \right) = \underline{\underline{488 \text{ kHz}}}$$

$$\% \text{ mod} = \frac{488}{75} (100) = \underline{\underline{651 \% \text{ mod}}}$$

(b.) The amplitudes of the high frequency audio components are much smaller than those of the low frequency components. For example, if the components at 15 KHz are more than $20 \log_{10} \left(\frac{7.21}{1.1} \right) = 16.3$ dB below the 1 KHz components, there is no problem.



7-49. Cont'd (b.) Using Fig. P7-49, it is seen that the signal power at point ① is $1/5$ the signal power at ②.

$$S_1 = \overline{m_1^2} = \frac{1}{5} \overline{m_b^2} = \frac{2}{5} P_s$$

The noise power at point ① is :

$$N_1 = N_0 [2(4\text{kHz})] = (8 \times 10^3) N_0$$

$$\Rightarrow \left(\frac{S}{N}\right)_1 = \frac{\frac{2}{5} P_s}{(8\text{k}) N_0} = \frac{P_s}{(20 \times 10^3) N_0} = (5 \times 10^{-5}) \frac{P_s}{N_0}$$

Also for SSB $(S/N)_{\text{out}} = (S/N)_{\text{in}}$

$$\therefore \underline{\left(\frac{S}{N}\right)_{o1} = (5 \times 10^{-5}) P_s / N_0} ; \text{ the same result is obtained for the other four channels.}$$