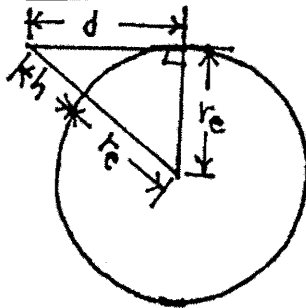


Chapter 1

1-3



$$(h+r_e)^2 = d^2 + r_e^2$$

$$\Rightarrow h^2 + 2hr_e = d^2 \text{ where } h \ll r_e$$

$$\Rightarrow d^2 \approx 2hr_e \text{ and } r_e = \frac{4}{3}(3960 \text{ miles}) = 5280 \text{ miles}$$

Let h = antenna height in feet, and d in miles.

$$\Rightarrow d^2 (\text{miles})^2 = 2h (\text{feet}) \left(\frac{1 \text{ mile}}{5280 \text{ feet}} \right) (5280 \text{ miles})$$

$$\Rightarrow d^2 = 2h \text{ or } \underline{d = \sqrt{2h}}$$

1-6

$$\{n_i\} = \{-1.0, 0.0, 3.0, 4.0\} \quad i=1, 4; \quad p_1=p_2=0.2; \quad p_3=p_4=0.3$$

$$\Rightarrow I_1=I_2 = \frac{-\ln(0.2)}{\ln 2} = 2.322 \text{ bits}, \quad I_3=I_4 = \frac{-\ln(0.3)}{\ln 2} = 1.737 \text{ bits}$$

$$H = \sum_{j=1}^M p_j I_j = 2[(0.2)(2.322) + (0.3)(1.737)] = \underline{1.971 \text{ bits}}$$

1-9

Let p_1 = prob. of sending a binary 1

(a) p_2 = prob. of sending a binary 0 = $1-p_1$

$$H = \sum_{i=1}^2 p_i I_i = p_1 \log_2 \left(\frac{1}{p_1} \right) + (1-p_1) \log_2 \left(\frac{1}{1-p_1} \right)$$

$$H = \frac{1}{\ln 2} [-p_1 \ln(p_1) - (1-p_1) \ln(1-p_1)]$$

$$\frac{\partial H}{\partial p_1} = 0 \Rightarrow -(\ln p_1 + 1) - (-1) \ln(1-p_1) + \frac{1-p_1}{1-p_1} (-1) = 0$$

$$\Rightarrow -\ln p_1 - 1 + \ln(1-p_1) + 1 = 0$$

$$\text{or } \ln \left(\frac{1-p_1}{p_1} \right) = 0 = \ln 1$$

$$\text{thus } \frac{1}{p_1} - 1 = 1 \Rightarrow \underline{p_1 = \frac{1}{2} = p_2}$$

$$(b) H_{\max} = \frac{1}{2} \log_2 2 + \left(1 - \frac{1}{2}\right) \log_2 2 = \underline{1 \text{ bit}}$$

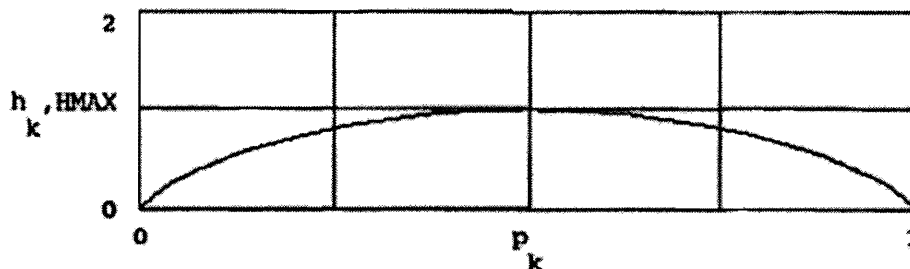
Math CAD Solution

LET p = The probability for sending a binary 1, then the probability for sending a binary 0 is $(1-p)$. From the entropy formula for $H(p)$, we can draw the figure of $H(p)$, and from this figure, we can find the maximum entropy and the p .

$$H(p) = (p \cdot \ln(p) + (1-p) \cdot \ln(1-p)) / (-\ln(2))$$

$$k = 0 \dots 50 \quad p_k := \frac{k}{50} \quad HMAX := 1$$

$$h_k := \frac{-1}{\ln(2)} \left[p_k \ln[p_k] + [1 - p_k] \cdot \ln[1 - p_k] \right]$$



From the above figure, we know the maximum entropy is 1 where the probability for sending 1 or 0 is 0.5.

1-10

$$P_1 = 0.25 ; P_2 = P_3 = 0.15 ;$$

$$P_4 = P_5 = P_6 = P_7 = P_8 = P_9 = 0.07$$

$$0.25 + 2(0.15) + 6(0.07) = 0.97$$

$$\therefore P_{10} = 0.03 \quad \text{since} \quad \sum_{j=1}^{10} P_j = 1.0$$

$$H = \sum_{j=1}^m P_j \log_2 \left(\frac{1}{P_j} \right) = \left[\frac{-1}{\ln 2} \right] \sum_{j=1}^{10} P_j \ln P_j$$

$$= \left[\frac{-1}{\ln 2} \right] \left[.25 \ln .25 + (2) .15 \ln .15 \right. \\ \left. + (6) .07 \ln .07 + .03 \ln .03 \right]$$

$$\underline{\underline{H = 3.084 \text{ bits}}}$$

1-12

$$M = 10 \quad P_i = \frac{1}{10} \quad i = 1, 10 \quad R = \frac{H}{T} = 3 \frac{b}{s}$$

$$H = \frac{-10(.1) \ln .1}{\ln 2} = 3.322 \text{ bits}$$

$$T = \frac{H}{R} = \frac{3.322 \text{ bits}}{3 \text{ bits/sec}} = \underline{\underline{1.11 \text{ sec.} = T}}$$

1-15

(a) chars := 110 Number of characters available

$$b := \text{ceil} \left[\frac{\log(\text{chars})}{\log(2)} \right] \quad \text{Number of bits required to represent a character}$$

$$\longrightarrow b = 7 \quad \text{bits}$$

(b) B := 3200 Hz Channel bandwidth
SNRdB := 20 dB Signal to noise ratio

$$\text{SNR} := 10^{\frac{\text{SNRdB}}{10}} \longrightarrow \text{SNR} = 100 \quad (\text{Absolute power ratio})$$

$$C := B \cdot \left[\frac{\log(1 + \text{SNR})}{\log(2)} \right] \Rightarrow C = 2.131 \cdot 10^4 \quad \text{Channel capacity (bits/sec)}$$

$$C := \frac{C}{b} \longrightarrow C = 3.044 \cdot 10^3 \quad \text{Channel capacity (chars/sec)}$$

(c) Assuming equally likely characters,
information content of each character is:

$$P := \frac{1}{\text{chars}} \quad \text{Probability of each character}$$

$$I := \frac{\log \left[\frac{1}{P} \right]}{\log(2)} \longrightarrow I = 6.781 \quad \text{bits}$$

1-18

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x0 := 1  x1 := 0  x2 := 1  x3 := 1  x4 := 1  Input vector
ga0 := 1  ga1 := 0  ga2 := 0  ga3 := 1  Gain vector, mod2 adder
gb0 := 1  gb1 := 1  gb2 := 1  gb3 := 1  Gain vector, mod2 adder

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k := 0 ..length(ga) - 2
v := 0  k := 0 ..length(x) - 1  vk+length(ga)-1 := xk
k := length(x) + length(ga) - 1 ..length(x) + 2 length(ga) - 3
vk := 0  i := 0 ..length(v) - length(ga)
j := 0 ..length(ga) - 1

```

$$sa_i := \sum_j [ga_{\text{length}(ga)-j-1} v_{j+i}]$$

$$sa_i := \text{mod}[sa_i, 2]$$

$$sb_i := \sum_j [gb_{\text{length}(gb)-j-1} v_{j+i}]$$

$$sb_i := \text{mod}[sb_i, 2] \quad s_{2i} := sa_i \quad s_{2i+1} := sb_i$$

$$i := 0 ..2 \text{ length}(x) - 1$$

$$out_i := s_i$$

For $x^T = (1 \ 0 \ 1 \ 1 \ 1)$
 =====> $out^T = (1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 1 \ 1)$