Chapter 7

7-1 (a.)

$$r_0 = \begin{cases} A + n_0 \ , & s_1 \text{ sent} \\ -A + n_0 \ , & s_2 \text{ sent} \end{cases}$$

$$\Rightarrow f(r_0|s_1) = \frac{1}{\sqrt{2}\,\Delta_0} e^{-\frac{\sqrt{2}\,|r_0 - A|}{\Delta_0}}$$

$$f(r_0|s_2) = \frac{1}{\sqrt{2}\,\Delta_0} e^{-\frac{\sqrt{2}\,|r_0 + A|}{\Delta_0}}$$

Using (7-8)

$$P_e = \frac{1}{2}\int_{-\infty}^{V_T} \frac{1}{\sqrt{2}\,\Delta_0} e^{-\frac{\sqrt{2}\,|r_0 - A|}{\Delta_0}} dr_0 + \frac{1}{2}\int_{V_T}^{\infty} \frac{1}{\sqrt{2}\,\Delta_0} e^{-\frac{\sqrt{2}\,|r_0 + A|}{\Delta_0}} dr_0$$

$m_{r_{01}} = A$, $m_{r_{02}} = -A$, the source probabilities are equally likely, and the conditional probabilities have symmetrical shapes about $\pm A$.

Thus $\boxed{V_T = 0}$.

$$\therefore P_e = \frac{1}{2\sqrt{2}\,\Delta_0}\left[\int_{-\infty}^{0} e^{-\frac{\sqrt{2}\,|r_0 - A|}{\Delta_0}} dr_0 + \int_{0}^{\infty} e^{-\frac{\sqrt{2}\,|r_0 + A|}{\Delta_0}} dr_0\right]$$

$$= \frac{1}{2\sqrt{2}\,\Delta_0}\left[\int_{-\infty}^{0} e^{\frac{\sqrt{2}(r_0 - A)}{\Delta_0}} dr_0 + \int_{0}^{\infty} e^{-\frac{\sqrt{2}(r_0 + A)}{\Delta_0}} dr_0\right]$$

Let $x_1 = \frac{\sqrt{2}(r_0 - A)}{\Delta_0}$ and $x_2 = \frac{-\sqrt{2}(r_0 + A)}{\Delta_0}$

$dx_1 = \frac{\sqrt{2}}{\Delta_0} dr_0$ $\qquad dx_2 = \frac{-\sqrt{2}}{\Delta_0} dr_0$

$$= \frac{1}{2\sqrt{2}\,\Delta_0}\left[\int_{-\infty}^{-\sqrt{2}A/\Delta_0} e^{x_1}\left(\frac{\Delta_0}{\sqrt{2}} dx_1\right) + \int_{-\sqrt{2}A/\Delta_0}^{-\infty} e^{x_2}\left(\frac{-\Delta_0}{\sqrt{2}} dx_2\right)\right]$$

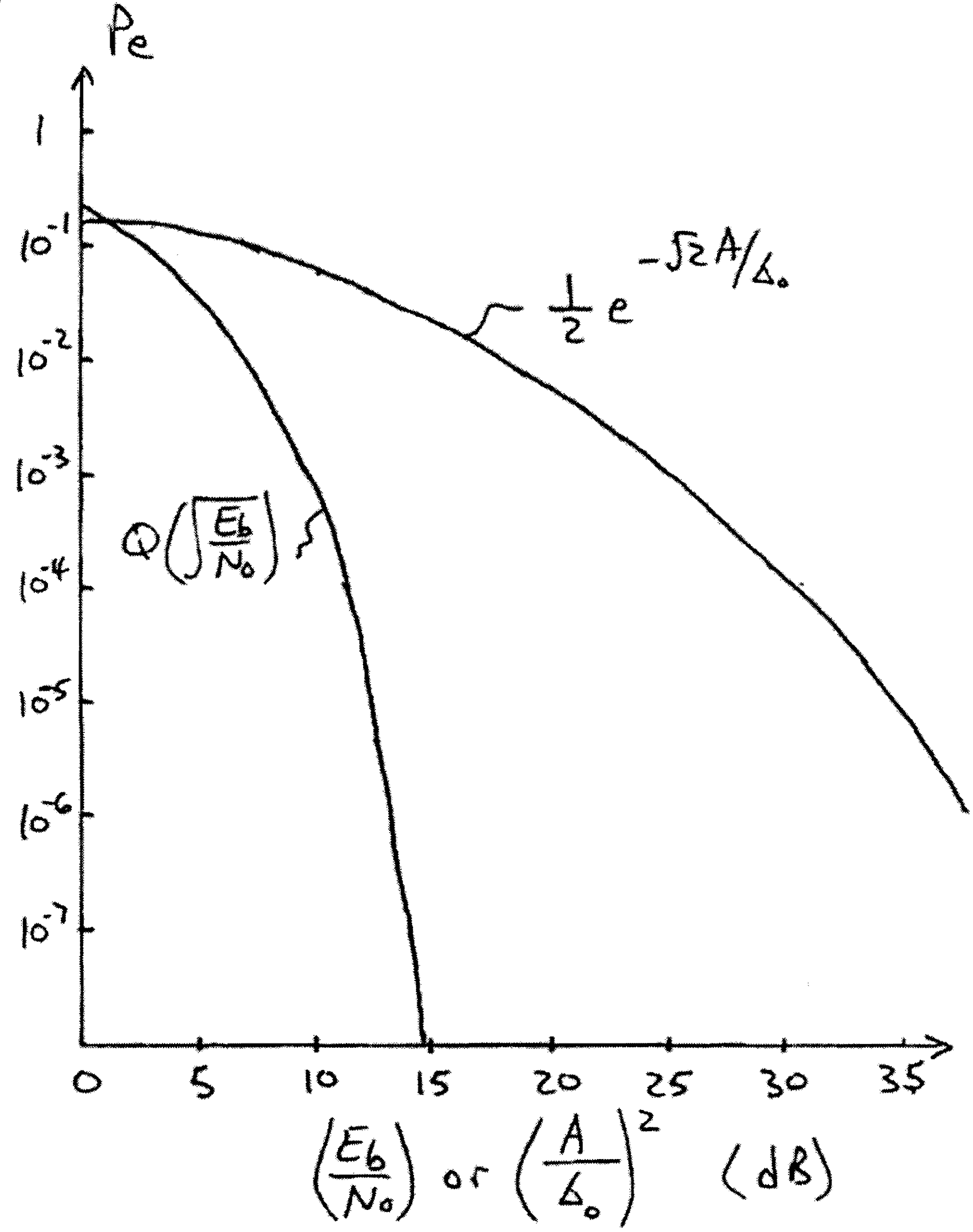
$$= \frac{1}{4}\left[\int_{-\infty}^{-\sqrt{2}A/\Delta_0} e^{x_1} dx_1 + \int_{-\infty}^{-\sqrt{2}A/\Delta_0} e^{x_2} dx_2\right]$$

7-1. (a.) Cont'd

$$P_e = \frac{1}{2}\left[\int_{-\infty}^{-\sqrt{2}A/\Delta_0} e^{x}\,dx\right] = \frac{1}{2}\, e^{x}\Big|_{-\infty}^{-\sqrt{2}A/\Delta_0}$$

$$= \frac{1}{2}\left\{e^{-\sqrt{2}A/\Delta_0} - e^{-\infty}\right\} = \underline{\underline{\frac{1}{2}\, e^{-\sqrt{2}A/\Delta_0}}} = P_e$$

(b.)



P_e <u>much</u> larger for Laplacian Noise.

7-6

$$\left(\frac{S}{N}\right)_{in} = \frac{\frac{E_b}{T_b}}{\left(\frac{N_o}{2}\right)(2B_{eq})} = \frac{E_b R}{N_0 B_{eq}} \Rightarrow \frac{E_b}{N_0} = \frac{B_{eq}}{R}\left(\frac{S}{N}\right)_{in}$$

Aside:

$$B_{eq} = \frac{\int_{-\infty}^{\infty} |H(f)|^2 df}{2|H(0)|^2} = \frac{\cancel{\left(\frac{2K}{N_0}\right)^2} \int_{-\infty}^{\infty} |S(f)|^2 df}{2\cancel{\left(\frac{2K}{N_0}\right)^2} |S(0)|^2} = \frac{\int_{-\infty}^{\infty} |S(f)|^2 df}{2|S(0)|^2}$$

Using (6-155) for MF: $H(f) = \frac{K S^*(f) e^{-j\omega t_0}}{N_0/2}$

$$\Rightarrow B_{eq} = \frac{\cancel{T_b^2} \int_{-\infty}^{\infty} \left(\frac{\sin(\pi T_b f)}{\pi T_b f}\right)^2 df}{2\cancel{T_b^2}\, 1^2} = \frac{1}{2\pi T_b}\int_{-\infty}^{\infty}\left(\frac{\sin x}{x}\right)^2 dx = \frac{1\ \pi}{2\pi T_b} = \frac{1}{2}R$$

$s(t) = \Pi\left(\frac{t}{T_b}\right) \Leftrightarrow S(f) = T_b \frac{\sin(\pi T_b f)}{\pi T_b f}$

Let $x = \pi T_b f$
$dx = \pi T_b df$

$$\Rightarrow \frac{E_b}{N_0} = \frac{\frac{1}{2}\cancel{R}}{\cancel{R}}\left(\frac{S}{N}\right)_{in} = \frac{1}{2}\left(\frac{S}{N}\right)_{in} = \frac{E_b}{N_0}$$

Using (7-24b):

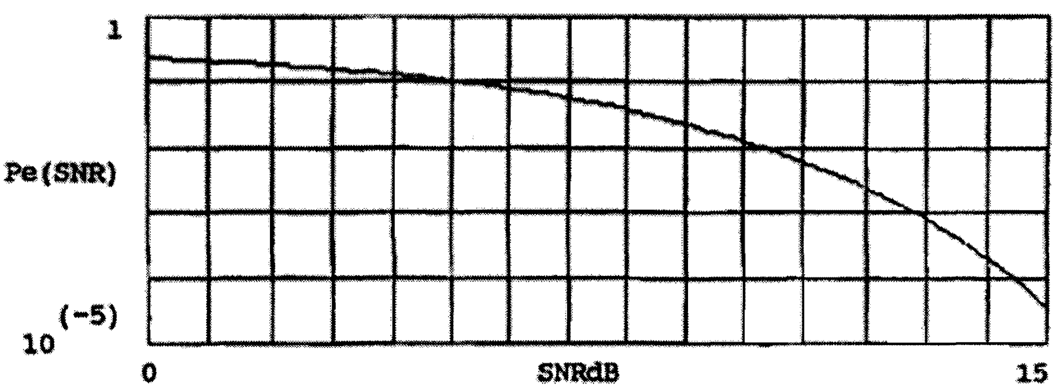
$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{1}{2}\left(\frac{S}{N}\right)_{in}}\right)$$

SNRdB := 0,0.1 ..15

Q(x) := 1 − cnorm(x)

$SNR(SNRdB) := 10^{\frac{SNRdB}{10}}$

$Pe(SNR) := Q\left[\sqrt{0.5 \cdot SNR(SNRdB)}\right]$



7-9 (a.) For derivation of P_e, follow the same procedure as used in the solution for Prob. 7-8.

$$B_{eq} = \int_0^\infty \frac{|H(f)|^2}{|H(0)|^2}\,df = \int_0^\infty \frac{1}{1+\left(\frac{f}{f_0}\right)^4}\,df = f_0 \int_0^\infty \frac{1}{1+x^4}\,dx = \frac{f_0 \pi}{2\sqrt{2}}$$

Let $x = f/f_0$ Using Sec. A-3

$$s_{01} = \int_0^T s_{01}(T-\lambda)\,h(\lambda)\,d\lambda = \int_0^T A\left[\sqrt{2}\,\omega_0 e^{-(\omega_0/\sqrt{2})\lambda} \sin\left(\frac{\omega_0}{\sqrt{2}}\lambda\right)\right] d\lambda$$

or

$$s_{01} = \int_0^T 2A\, e^{-x} \sin(x)\,dx = A\left[1 - e^{-\sqrt{2}\pi}\left(\sin(\sqrt{2}\pi) + \cos(\sqrt{2}\pi)\right)\right]$$

Let $x = \frac{\omega_0}{\sqrt{2}}\lambda$ Using Sec. A-5

$$\Rightarrow \underline{s_{01} = 1.01447\,A}$$

$f_0 = \frac{1}{T}$

$$\sigma_0^2 = \frac{N_0}{\cancel{2}}\,\cancel{2}B_{eq} = N_0 B_{eq} = \frac{N_0 \pi f_0}{2\sqrt{2}} = \frac{N_0 \pi}{2\sqrt{2}\,T}$$

Using (7-17) where $s_{01} = -s_{02}$,

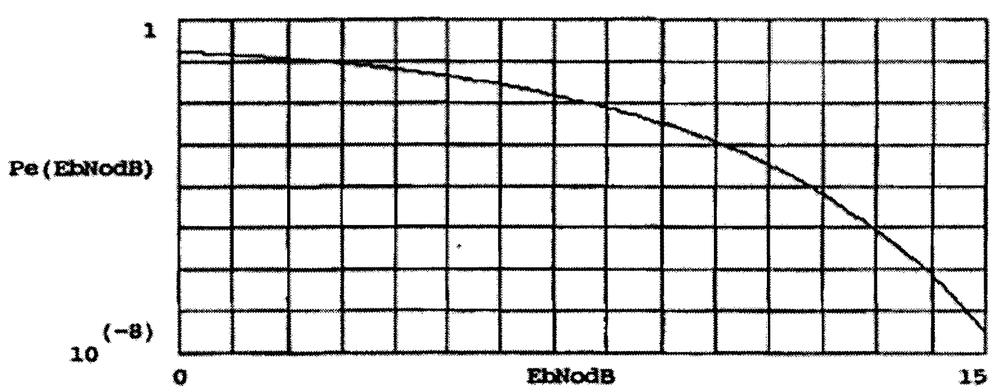
$$P_e = Q\left(\sqrt{\frac{s_{01}^2}{\sigma_0^2}}\right) = Q\left(\sqrt{\frac{(1.01447)^2 A^2}{\frac{N_0 \pi}{2\sqrt{2}\,T}}}\right) = Q\left(\sqrt{\frac{2\sqrt{2}(1.01447)^2 A^2 T}{\pi N_0}}\right)$$

$$\Rightarrow P_e = Q\left(\sqrt{\frac{2\sqrt{2}(1.01447)^2}{\pi}\left(\frac{E_b}{N_0}\right)}\right) = \underline{\underline{Q\left(\sqrt{0.92656\left(\frac{E_b}{N_0}\right)}\right)}}$$

(b.)

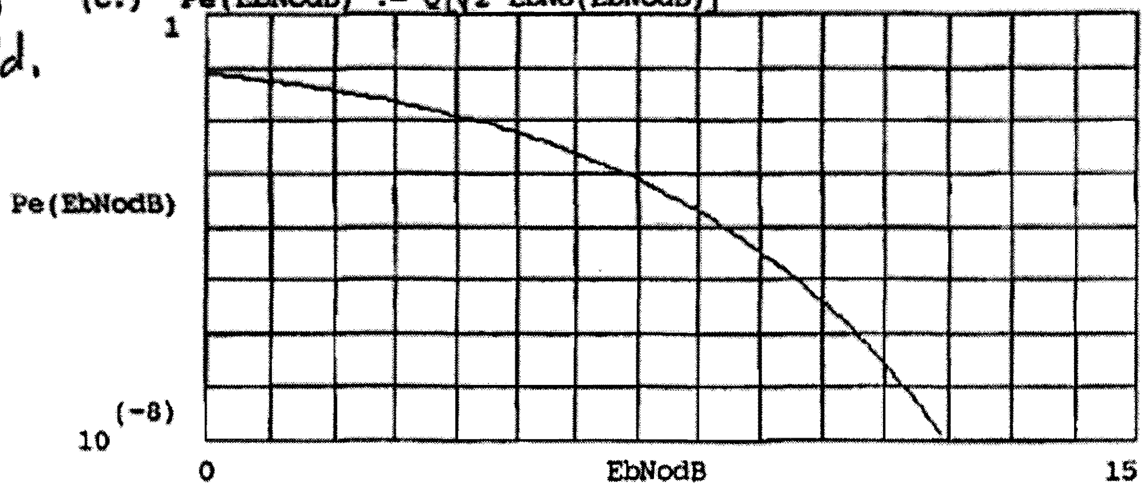
```
EbNodB := 0,0.1 ..15
Q(x) := 1 - cnorm(x)
                                    EbNodB
                                    ------
                                      10
EbNo(EbNodB) := 10

Pe(EbNodB) := Q[√(0.92656·EbNo(EbNodB))]
```



7-9 Cont'd.

(c.) $Pe(EbNodB) := Q\left[\sqrt{2 \; EbNo(EbNodB)}\right]$



7-12

(a.) Referring to the solution for SA 7-3,

$$P_e = Q\left(\sqrt{\frac{A^2}{4N_0B}}\right) \qquad (7\text{-}24a)$$

where $B = \frac{2}{T} = 2R$ and $E_b = \left(\frac{A^2}{2}\right)T = \frac{A^2}{2R}$

Thus,
$$P_e = Q\left(\sqrt{\frac{A^2}{8N_0R}}\right) = Q\left(\sqrt{\frac{A^2}{4N_0 2R}}\right) = Q\left(\sqrt{\frac{1}{4}\left(\frac{E_b}{N_0}\right)}\right)$$

7-12 Cont'd

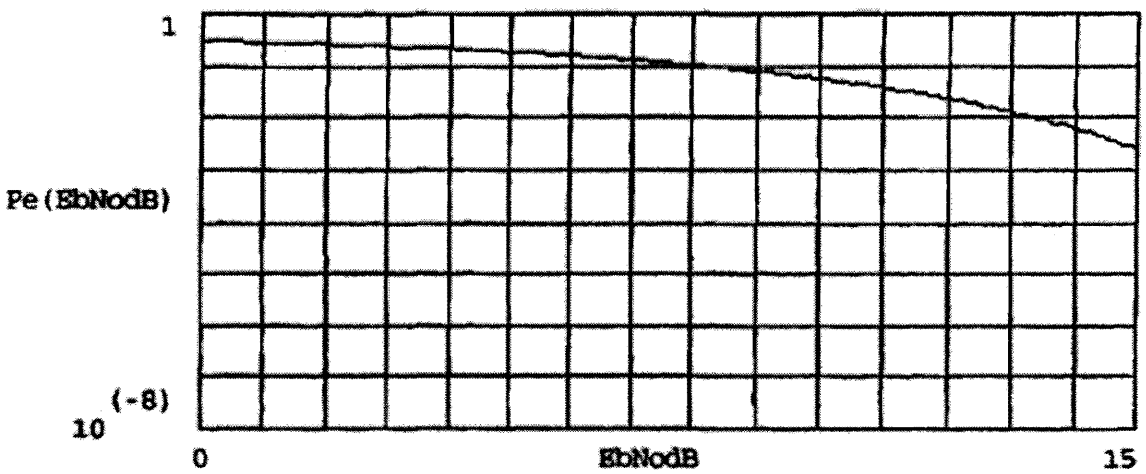
EbNodB := 0,0.1 ..15

Q(x) := 1 - cnorm(x)

$$EbNo(EbNodB) := 10^{\frac{EbNodB}{10}}$$

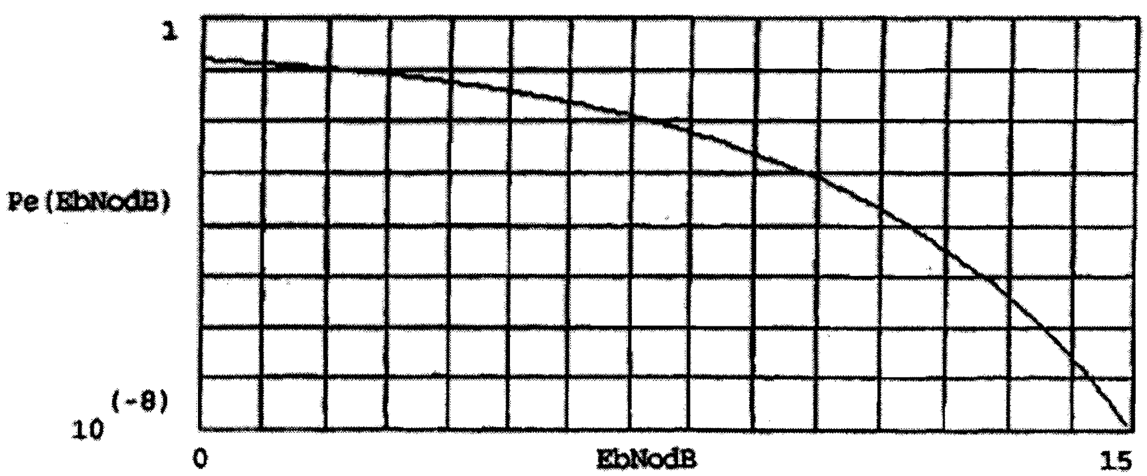
(a.)

$$Pe(EbNodB) := Q\left[\sqrt{0.25\cdot EbNo(EbNodB)}\right]$$ <---- LPF with ISI Result



(b.)

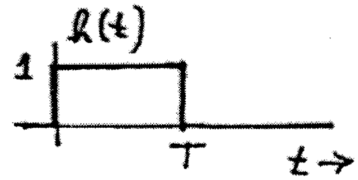
$$Pe(EbNodB) := Q\left[\sqrt{EbNo(EbNodB)}\right]$$ Matched Filter Result <-- (7-24b)



7-15

(a) The impulse response is:

$$h(t) = s_{01}(T-t) = s_{01}(t)$$



$$H(f) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt = \int_0^T e^{-j\omega t}\, dt = \frac{e^{-j\omega t}}{j\omega}\Big|_0^T = \frac{e^{-j\omega T} - e^{-j(0)}}{-j\omega}$$

$$\Rightarrow H(f) = e^{-j\omega T/2}\left[\frac{e^{j\omega T/2} - e^{-j\omega T/2}}{j2\pi f}\right] = Te^{-j\pi f T}\left[\frac{\sin(\pi f T)}{\pi f T}\right]$$

See Fig. 6-17

(b)

$$B_{eq} = \frac{\int_0^{\infty} |H(f)|^2 df}{|H(0)|^2} = \frac{T^2 \int_0^{\infty} \left[\frac{\sin(\pi f T)}{\pi f T}\right]^2 df}{T^2}$$

$$= \int_0^{\infty} \left(\frac{\sin x}{x}\right)^2 \left(\frac{1}{\pi T} dx\right) = \frac{1}{\pi T}\left(\frac{\pi}{2}\right) = \frac{1}{2T} = B_{eq}$$

Let $x = \pi f T$; $dx = \pi T\, df$

Using Sec. A-5

7-17 From (7-8)

$$P_e = P(1)\int_{-\infty}^{V_T} f(r_0|s_1)\, dr_0 + P(0)\int_{V_T}^{\infty} f(r_0|s_2)\, dr_0$$

where
$$r_0 = \begin{cases} A + n_0, & \text{for a binary 1 sent} \\ -A + n_0, & \text{" 0 "} \end{cases}$$

Thus

$$P_e = P(1)\int_{-\infty}^{V_T} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(r_0 - A)^2}{2\sigma^2}} dr_0 + P(0)\int_{V_T}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(r_0 + A)^2}{2\sigma^2}} dr_0$$

Let $\lambda_1 = -(r_0 - A)/\sigma$; $\lambda_2 = (r_0 + A)/\sigma$

$d\lambda_1 = -\frac{1}{\sigma} dr_0$ $\qquad$ $d\lambda_2 = \frac{1}{\sigma} dr_0$

7-17 Cont'd

$$\Rightarrow P_e = P(1)\int_{(-V_T+A)/\sigma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\lambda_1^2/2} d\lambda_1 + P(0)\int_{(V_T+A)/\sigma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\lambda_2^2/2} d\lambda_2$$

$$\text{or } P_e = P(1)\,Q\left[\frac{-V_T+A}{\sigma}\right] + P(0)\,Q\left[\frac{V_T+A}{\sigma}\right]$$

$$= P(1)\,Q\left[\sqrt{\frac{(-V_T+A)^2\,2T}{N_0}}\right] + P(0)\,Q\left[\sqrt{\frac{(V_T+A)^2\,2T}{N_0}}\right]$$

$$\sigma^2 = \frac{N_0}{2T}$$

To Check : Let $P(1) = P(0) = \frac{1}{2}$; $V_T = 0$

$$P_e = \frac{1}{2}\,Q\left[\sqrt{\frac{A^2\,2T}{N_0}}\right] + \frac{1}{2}\,Q\left[\sqrt{\frac{A^2\,2T}{N_0}}\right]$$

$$= Q\left[\sqrt{\frac{2A^2T}{N_0}}\right]$$; This checks with eqn. (7-26b)

7-21 Referring to Fig. 7-7

(a.) Let $s_1(t) = A\cos\omega_c t$, $s_2(t) = -A\cos\omega_c t$

$n(t) = x(t)\cos\omega_c t - y(t)\sin\omega_c t$

Coherent reference $= 2\cos(\omega_c t + \theta_e)$

$$\Rightarrow r_0(t) = \pm A\cos\theta_e + n_0(t) = s_{0_1}(t) + n_0(t)$$

(A-11) where $n_0(t) = x(t)\cos\theta_e + y(t)\sin\theta_e$

$$\overline{n_0^2(t)} = \overline{x^2(t)}\cos^2\theta_e + 2\,\overline{x(t)\,y(t)}\cos\theta_e\sin\theta_e + \overline{y^2(t)}\sin^2\theta_e$$

$\overline{x(t)\,y(t)} = 0$ (6-133k)

$$\Rightarrow \overline{n_0^2(t)} = 2N_0B(\cos^2\theta_e + \sin^2\theta_e) = 2N_0B$$

$\overline{x^2} = \overline{y^2} = 2N_0B$

Using (7-17) : $H(f) =$ LPF

Corresponds to (7-36)

$$P_e = Q\left[\pm\sqrt{\frac{(s_{01}-s_{02})^2}{4\sigma_0^2}}\right] = Q\left[\pm\sqrt{\frac{4A^2\cos^2\theta_e}{8N_0B}}\right] = Q\left[\pm\sqrt{\frac{A^2\cos^2\theta_e}{2N_0B}}\right]$$

where $\left\{\begin{array}{l} + \text{ is used when } s_{01} > s_{02} \Rightarrow |\theta_e| < \pi/2 \\ - \text{ is used when } s_{02} > s_{01} \Rightarrow \frac{\pi}{2} < |\theta_e| < \pi \end{array}\right\}$

7-21 Cont'd

(b.) If $H(f)$ is matched to the output of the multiplier, then

$$P_e = Q\left(\sqrt{\frac{E_d}{2N_0'}}\right) \quad \text{using (7-20)}$$

Where $N_0' = 2N_0$ and

$$E_d = \int_0^T [s_{01}(t) - s_{02}(t)]^2 dt = \int_0^T 4A^2 \cos^2\theta_e \, dt$$

or

$$E_d = 4A^2T\cos^2\theta_e = 8E_b\cos^2\theta_e$$

$$E_b = \tfrac{1}{2}E_{s_1} + \tfrac{1}{2}E_{s_2} = \tfrac{1}{2}\left(\tfrac{1}{2}A^2T\right) + \tfrac{1}{2}\left(\tfrac{1}{2}A^2T\right) = \tfrac{1}{2}A^2T$$

$$\Rightarrow \underline{\underline{P_e = Q\left[\sqrt{2\left(\frac{E_b}{N_0}\right)\cos^2\theta_e}\right]}} \leftarrow \text{Corresponds to (7-38)}$$

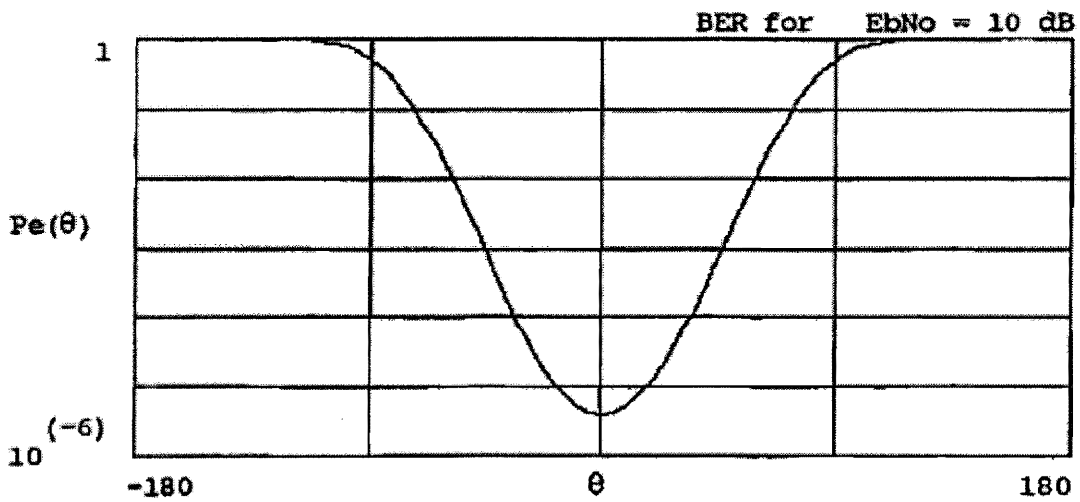
(c.) EbNodB := 10 θ := −180, −175 .. 180

$$\text{EbNo} := 10^{\frac{\text{EbNodB}}{10}}$$

Q(x) := 1 − cnorm(x)

$$z(\theta) := \sqrt{2\ \text{EbNo}\cdot\left[\cos\left[\pi\,\frac{\theta}{180}\right]\right]^2}$$

Pe(θ) := if(|θ| ≤ 90, Q(z(θ)), Q(−z(θ)))



7-22 (a.) Overall $P_e = 10\,(P_e)_i = \underline{\underline{5\times10^{-7}}}$

(b.) when repeaters were used, the E_b/N_o at the input to each was described by:

$$P_e = Q\left[\sqrt{2\left(\frac{E_b}{N_o}\right)}\right] = 5\times10^{-8} \underset{\text{Sec.A-10}}{\approx} \frac{1}{\sqrt{2\pi\left(\frac{2E_b}{N_o}\right)}}\, e^{-E_b/N_o}$$

$$\Rightarrow \frac{E_b}{N_o} = \underline{14.2}$$

Now with 10 amplifiers, the Rx input consists of the BPSK signal with E_b energy/bit plus a noise level 10 times that present before (since the line from one amp to the next contributes a PSD of $N_o/2$, and there are 10 such lines). Thus $\left(\frac{E_b}{N_o}\right)' = \frac{14.2}{10} = \underline{1.42}$

$$\therefore\ P_e' = Q\left(\sqrt{2(1.42)}\right) = Q(1.69) \approx Q(1.7) = \underline{\underline{4.4\times10^{-2}}}$$

7-23 (a.) $B_T = 2700 - 300 = 2400\text{ Hz}$ Band pass System

The largest bit rate that can be accomodated w/o I.S.I. is (Table 7-1.)

$$B = \underline{\underline{R = 2400}} \text{ bits/sec}$$

(b.) From Table 7.1 for BPSK:

$$P_e = Q\left(\sqrt{2\left(\frac{E_b}{N_o}\right)}\right) \underset{\text{Sec.A-10}}{\approx} \frac{1}{\sqrt{4\pi\left(\frac{E_b}{N_o}\right)}}\, e^{-E_b/N_o}$$, for each repeater

But $\left(\frac{S}{N}\right) = \frac{P_s}{N_o B_T} \underset{B_T=R}{=} \frac{P_s}{N_o R} \underset{R=\frac{1}{T}}{=} \frac{P_s T}{N_o} = \frac{E_b}{N_o} = \frac{S}{N} = 15\text{dB}$

7-23 Cont'd

$$\Rightarrow \frac{E_b}{N_o} = 15\,dB = 31.6$$

$$\therefore P_e = \frac{1}{\sqrt{4\pi(31.6)}} e^{-31.6} = 9.26 \times 10^{-16} / \text{repeater}$$

There are $n = \frac{600 \text{ mi.}}{50 \text{ mi/rep}} = 12$ repeaters (including R_x)

$$\therefore \text{Overall } (P_e) \approx n P_e = 12(9.26 \times 10^{-16}) = \underline{\underline{1.11 \times 10^{-14}}}$$

<u>Note</u>: If there are n repeaters, there is an error at the end of the line only if there are an <u>odd</u> number of errors along the line (for the bit in question).

$$\Rightarrow P(\underline{K} \text{ errors}) = \binom{n}{K} P_e^{K} (1-P_e)^{n-K} \; ; \; \binom{n}{K} = \frac{n!}{(n-K)!\,K!}$$

$$\Rightarrow \text{Overall } P_e = \sum_{\substack{K=1 \\ K \text{ odd}}}^{n} P(K \text{ errors}) \approx n P_e$$

If $nP_e \ll 1$

7-29 From Table 7-1 for FSK w/ noncoherent detection:

$$P_e = \frac{1}{2} e^{-\frac{1}{2}(E_b/N_{total})}$$

$$N_{total} = k(T_o + T_{eff}) = k(T_o + (F-1)T_o) = kFT_o$$

$T_{eff} = (F-1)T_o$

$$= (1.38 \times 10^{-23})(10^{6/10}) 290$$

$$\frac{E_b}{N_{total}} = \frac{P_s T}{N_{tot}} = \frac{P_s}{N_{tot} R} = \frac{V_s^2/R_A}{kFT_o R} = 28.53$$

$R = 1/T$

$$P_e = \frac{1}{2} e^{-\frac{1}{2}(28.53)} = \underline{\underline{3.2 \times 10^{-7}}}$$

7-33 See Table 7-1.

(a.) For QPSK the largest R and min P_e are obtained.

$$R = 2B = 2(2700-300) = \underline{\underline{4800 \text{ b/s}}}$$

$$\frac{S}{N} = \frac{P_s}{\frac{N_0}{2}(2B)} \underset{\uparrow}{=} \frac{2P_s}{N_0 R} \underset{\uparrow}{=} \frac{2P_sT}{N_0} = \frac{2E_b}{N_0}$$

$B = \frac{1}{2}R$ $\quad$ $R = \frac{1}{T}$

$$\Rightarrow \frac{E_b}{N_0} = \frac{1}{2}\left(\frac{S}{N}\right) = \frac{1}{2}\left(10^{2.5}\right) = 158.1 \Rightarrow 22 \text{ dB}$$

$$P_e = Q\left(\sqrt{2\left(\frac{E_b}{N_0}\right)}\right) \underset{\uparrow}{\ll} 10^{-5} \text{ for } \frac{E_b}{N_0} = 22\text{dB}$$

Figure 7-14.

$$C = B\log_2\left(1+\frac{S}{N}\right) = 2400\log_2\left(1+10^{2.5}\right)$$

$$= \frac{2400}{\ln 2}\ln(1+316.23) = 1.99\times 10^4 = \underline{\underline{19{,}900 \text{ b/s}}}$$

7-34 (a.) $R = \left(8K \dfrac{\text{samples}}{\text{sec}}\right)\left(8 \dfrac{\text{bits}}{\text{sample}}\right) = 64\ K\ b/s$

$$\frac{S}{N} = \frac{P_s}{N_o B} = \frac{P_s T_b}{N_o} = \frac{E_b}{N_o} = 10^{.8} = 6.3$$

$B = R = \dfrac{1}{T_b}$ ← Table 7-1. for DPSK :

$$P_e = \frac{1}{2} e^{-\left(\frac{E_b}{N_o}\right)} = \frac{1}{2} e^{-6.3} = \underline{\underline{9.18 \times 10^{-4}}}$$

(b.) Using (7-70) with $M = 2^8 = 256$:

$$\left(\frac{S}{N}\right)_{OUT} = \frac{3M^2}{1+4(M^2-1)P_e} = \frac{3(256)^2}{1+4[(256)^2-1]9.18\times 10^{-4}}$$

$$= 813.6 \Rightarrow \underline{\underline{29.1\ dB}}$$

7-36 Using (7-89),

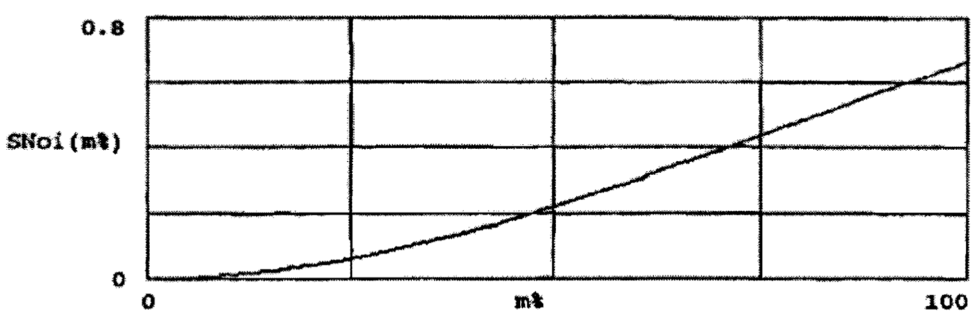
$$SN_{oi} \triangleq \frac{(S/N)_{out}}{(S/N)_{in}} = \frac{2\overline{m^2}}{1+\overline{m^2}}$$

Let $m(t) = \left(\frac{m\%}{100}\right)\cos \omega_m t \Rightarrow \overline{m^2} = \frac{1}{2}\left(\frac{m\%}{100}\right)^2$

$$\Rightarrow SN_{oi} = \frac{\left(\frac{m\%}{100}\right)^2}{1+\frac{1}{2}\left(\frac{m\%}{100}\right)^2}$$

m% := 0,5 ..100

$$SNoi(m\%) := \frac{\left[\frac{m\%}{100}\right]^2}{1 + 0.5\left[\frac{m\%}{100}\right]^2}$$



7-37 $m(t) = 0.4 \sin \omega_a t \Rightarrow \overline{m^2} = \frac{(0.4)^2}{2} = 0.08$

For AM, with product detector, use (7-90).

$$\frac{(S/N)_{out}}{(S/N)_{base}} = \frac{\overline{m^2}}{1+\overline{m^2}} = \frac{0.08}{1.08} = \underline{\underline{0.0741 \Rightarrow -11.3\,dB}}$$

Also get same result for env. det when (S/N) is large.

For DSB-SC, use (7-98).

$$\frac{(S/N)_{out}}{(S/N)_{base}} = \underline{\underline{1 \Rightarrow 0\,dB}} \Rightarrow \underline{\underline{\text{The AM system is inferior by } 11.3\,dB}}$$

7-44 Using Carson's Rule:

$B_{IF} = 2(\beta_f + 1)B \Rightarrow 25\,KHz = 2(\beta_f+1)5\,KHz \Rightarrow \beta_f = 1.5$

$f_1 = 2.1\,KHz$; $\frac{B}{f_1} = \frac{5}{2.1} \not\gg 1$ $\therefore$ (7-139) is not valid

Eqn. (7-124a) $s_o(t) = \frac{K D_f}{2\pi} m(t) = \frac{K B \beta_f}{V_p} m(t)$

$\overline{s_o^2(t)} = K^2 B^2 \beta_f^2 \overline{\left(\frac{m}{V_p}\right)^2}$; $\overline{\left(\frac{m}{V_p}\right)^2} = \frac{1}{2}$ for sinusoid

Eqn. (7-136) $\overline{[\tilde{n}_o(t)]^2} = 2\left(\frac{K}{A_c}\right)^2 N_o f_1^3 \left[\frac{B}{f_1} - \tan^{-1}\left(\frac{B}{f_1}\right)\right]$

$\therefore$

$$\left(\frac{S}{N}\right)_o = \frac{K^2 B^2 \beta_f^2 \overline{\left(\frac{m}{V_p}\right)^2}}{2\left(\frac{K}{A_c}\right)^2 N_o f_1^3 \left[\frac{B}{f_1} - \tan^{-1}\left(\frac{B}{f_1}\right)\right]}$$

7-44 Cont'd

Eqn. (7-128) $\left(\frac{S}{N}\right)_{in} = \frac{A_c^2}{4N_o(\beta_f+1)B}$

$$\frac{(S/N)_o}{(S/N)_{in}} = \frac{2\left(\frac{B}{f_1}\right)^3 \beta_f^2(\beta_f+1)\left(\overline{\frac{m}{V_p}}\right)^2}{\left[\frac{B}{f_1} - \tan^{-1}\left(\frac{B}{f_1}\right)\right]} = \frac{2(13.5)(2.25)(2.5)\frac{1}{2}}{1.21} = 62.9$$

$N_{total} = kFT_o \quad ; \quad F = 10^{1.2} = 15.8$

$$\left(\frac{S}{N}\right)_{in} = \frac{P_s}{\frac{N_{tot}}{2}(2B_{IF})} = \frac{P_s}{kFT_oB_{IF}} = \frac{P_s}{1.57\times10^{-15}}$$

$$\Rightarrow P_s = \left(\frac{S}{N}\right)_o\left(\frac{1}{62.9}\right)(1.57\times10^{-15})$$

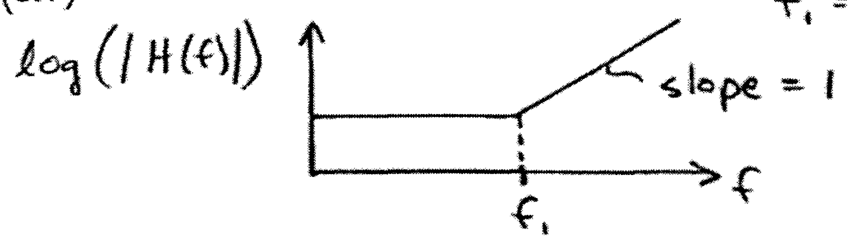
$$= 10^{3.5}\left(\frac{1}{62.9}\right)(1.57\times10^{-15}) = 7.89\times10^{-14}\ \text{W}$$

$$10\log_{10}\left(\frac{7.89\times10^{-14}}{10^{-3}}\right) = \underline{\underline{-101\ \text{dBm}}} = P_{s\,min}$$

7-49 $H(f) = [1 + j\,f/f_1]$ for preemphasis

$f_1 = 2.1$ kHz

(a.)



At $f = 15$ kHz the gain is:

$$|H(f)| = \left|1 + j\left(\frac{15}{2.1}\right)\right| = \sqrt{1+\left(\frac{15}{2.1}\right)^2} = 7.21$$

At $f = 1$ kHz:

$$|H(f)| = \left|1 + j\left(\frac{1}{2.1}\right)\right| = \sqrt{1+\left(\frac{1}{2.1}\right)^2} = 1.10$$

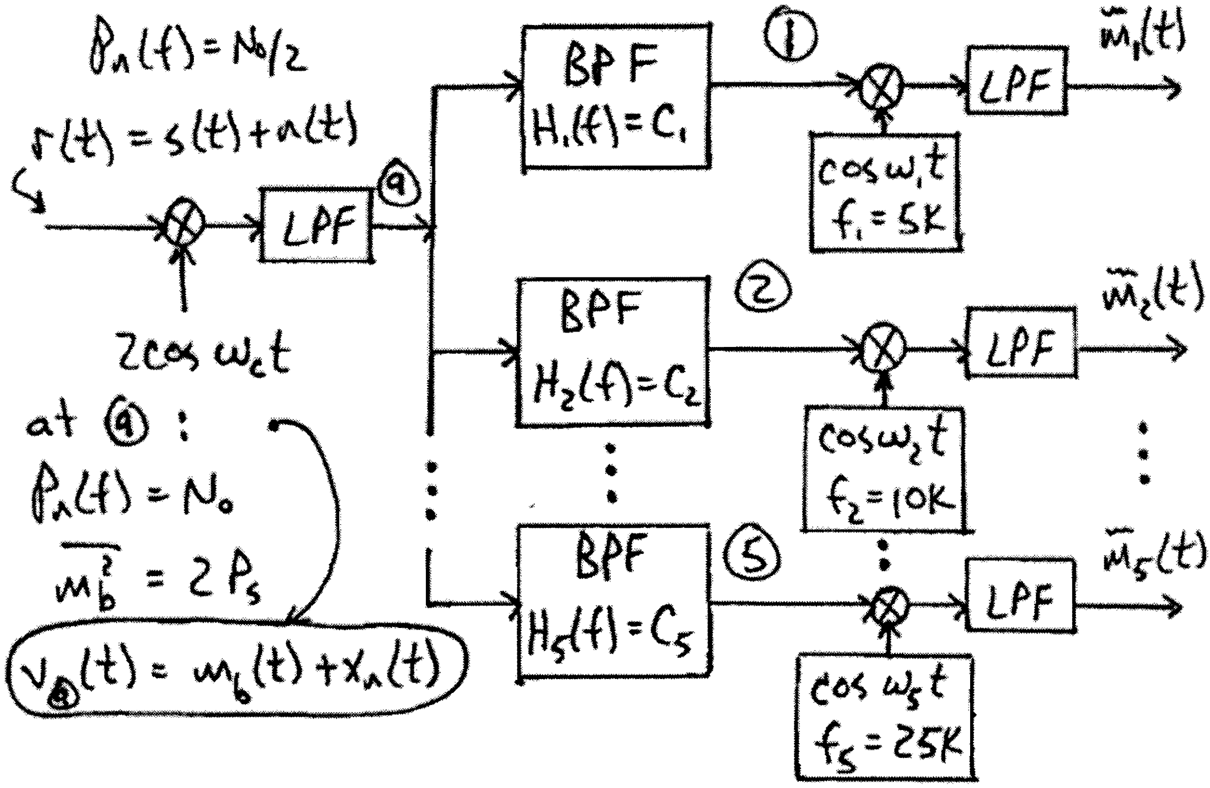
$$\Delta F = 75\text{ kHz}\left(\frac{7.21}{1.10}\right) = \underline{\underline{488\text{ kHz}}}$$

$$\%\text{ mod} = \frac{488}{75}(100) = \underline{\underline{651\%\text{ mod}}}$$

7-49 Cont'd

(b.) The amplitudes of the high frequency audio components are much smaller than those of the low frequency components. For example, if the components at 15 KHz are more than $20\log_{10}\left(\frac{7.21}{1.1}\right) = 16.3$ dB below the 1 KHz components, there is no problem.

7-51 (a.) $s(t) = m_b(t)\cos\omega_c t$; $P_s = \frac{1}{2}\overline{m_b^2}$



7-51 Cont'd (b.) Using Fig. P7-51, it is seen that the signal power at point ① is 1/5 the signal power at ⓐ.

$$S_1 = \overline{m_1^2} = \frac{1}{5}\overline{m_b^2} = \frac{2}{5}P_s$$

The noise power at point ① is:

$$N_1 = N_0[2(4\text{KHz})] = (8\times10^3)N_0$$

$$\Rightarrow \left(\frac{S}{N}\right)_1 = \frac{\frac{2}{5}P_s}{(8K)N_0} = \frac{P_s}{(20\times10^3)N_0} = (5\times10^{-5})\frac{P_s}{N_0}$$

Also for SSB $(S/N)_{out} = (S/N)_{in}$

$$\therefore \left(\frac{S}{N}\right)_{01} = (5\times10^{-5})\, P_s/N_0$$

; the same result is obtained for the other four channels.