Appendix B

B-1 $P(1) = \dfrac{n_1}{n} = \dfrac{1428}{1428 + 2668} = \underline{\underline{0.3486}}$

B-2 Total # of outcomes $= 6(6) = 36$

(a.) $8 =: (2+6), (3+5), (4+4), (5+3), (6+2)$

$\Rightarrow \underline{\underline{P(8) = 5/36}}$

(b.) $5 =: (1+4), (2+3), (3+2), (4+1) \Rightarrow P(5) = \dfrac{4}{36}$

$7 =: (1+6), (2+5), (3+4), (4+3), (5+2), (6+1)$

$\Rightarrow P(7) = \dfrac{6}{36}$

$P(5+7+8) = P(5) + P(7) + P(8)$

Mutually exclusive

$= \dfrac{1}{36}[4 + 6 + 5] = \underline{\underline{15/36}}$

B-4 $P(1+3+5) = P(1) + P(3) + P(5) = \dfrac{3}{6} = \underline{\underline{\dfrac{1}{2}}}$

B-5 $P(4/E) = \dfrac{P(4 \cdot E)}{P(E)} = \dfrac{P(4)}{P(E)} = \dfrac{1/6}{1/2} = \underline{\underline{\dfrac{1}{3}}}$

B-9

$$P\left(-\frac{A}{4} \le y \le \frac{A}{4}\right) = \int_{-A/4}^{A/4} f(y)\,dy = 2\int_0^{A/4} f(y)\,dy$$

$$= -\frac{2}{A^2}\int_0^{A/4} (y-A)\,dy =$$

Let $x = y - A$; $dx = dy$

$$2\int_0^{A/4} f(y)\,dy = \frac{-2}{A^2}\int_{-A}^{-3A/4} x\,dx = -\frac{x^2}{A^2}\Big|_{-A}^{-3A/4}$$

$$= \frac{-A^2}{A^2}\left[\left(\frac{-3}{4}\right)^2 - 1\right] = \underline{\underline{0.4375}}$$

B-12

(a.) $\int_{-\infty}^{\infty} f(x)\,dx = 1 = \int_0^{\infty} k e^{-bx}\,dx$

$$= \frac{ke^{-bx}}{-b}\Big|_0^{\infty} = \frac{k}{b}\left[e^{-\infty} - e^0\right] = \frac{k}{b} = 1 \Rightarrow \underline{\underline{k = b}}$$

(b.) $f(x) = b e^{-bx}$

$$m = \bar{x} = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^{\infty} b x e^{-bx}\,dx$$

$$= b\left[\frac{-xe^{-bx}}{b}\Big|_0^{\infty} + \int_0^{\infty} \frac{e^{-bx}}{b}\,dx\right] = b\left[\frac{e^{-bx}}{-b^2}\Big|_0^{\infty}\right]$$

Let $u = x$ $\quad dv = e^{-bx}dx$
$du = dx$ $\quad v = -e^{-bx}/b$

$$\Rightarrow m = b\left[\frac{-1}{b}\left(e^{-\infty} - e^0\right)\right] = \underline{\underline{\frac{1}{b} = m}}$$

(c.) $\sigma^2 = \overline{x^2} - (\bar{x})^2$ where $\overline{x^2} = \int_{-\infty}^{\infty} x^2 b e^{-bx}\,dx$

Using Sec. A-5 $\overline{x^2} = be^{-bx}\left[\frac{x^2}{-b} - \frac{2x}{b^2} - \frac{2}{b^3}\right]\Big|_0^{\infty} = -be^{-0}\left(\frac{-2}{b^3}\right) = \frac{2}{b^2}$

$$\Rightarrow \sigma^2 = \frac{2}{b^2} - \left(\frac{1}{b}\right)^2 = \underline{\underline{\frac{1}{b^2} = \sigma^2}}$$

B-17

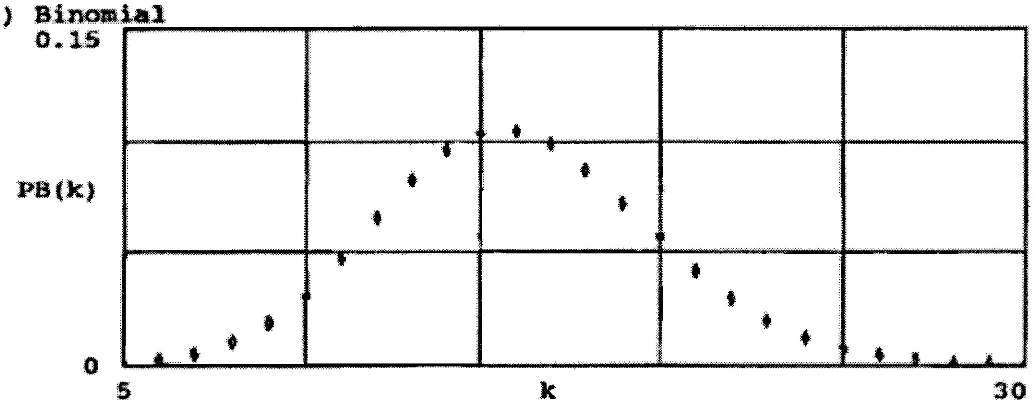
$n := 160 \qquad p := 0.1 \qquad q := 1 - p \qquad \lambda := n \cdot p$

$m := n\, p \qquad \sigma := \sqrt{n\, p \cdot q} \qquad k := 0\,..2\,\lambda \qquad m = 16$

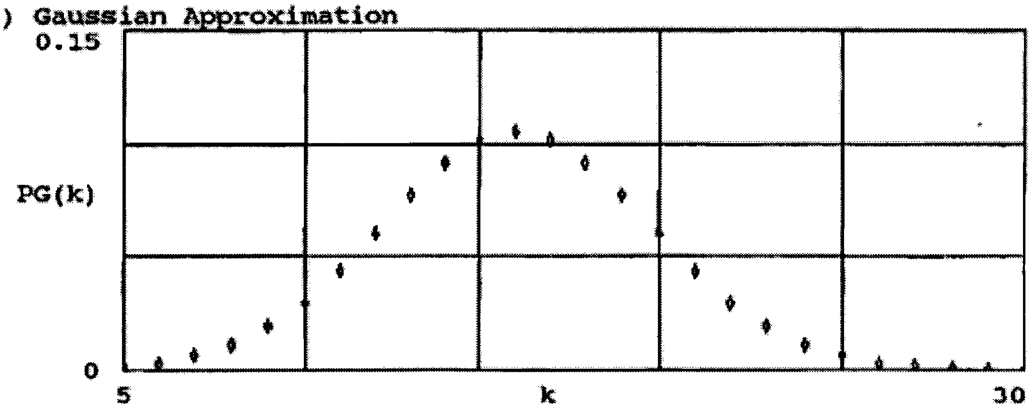
$$PB(k) := \frac{n!}{k!\,(n-k)!}\, p^{k}\, q^{n-k} \qquad \text{Binomial}$$

$$PP(k) := e^{-\lambda} \cdot \frac{\lambda^{k}}{k!} \qquad \text{Poisson} \qquad PG(k) := \frac{1}{\sigma \cdot \sqrt{2 \cdot \pi}}\, e^{\frac{-(k-m)^2}{2 \cdot \sigma^2}}$$

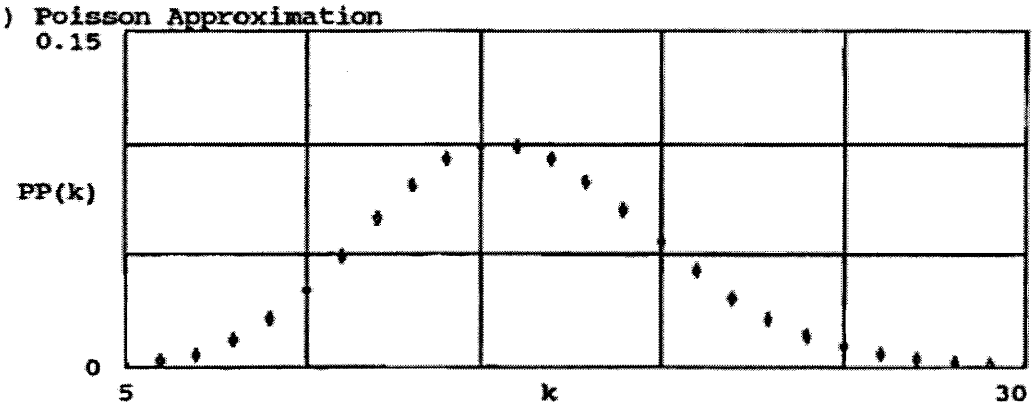
(a.) Binomial



(b.) Gaussian Approximation



(c.) Poisson Approximation



B-23 let $f(x)$ = pdf of R values where x = R value

$$\Rightarrow \int_{.9\bar{x}}^{1.1\bar{x}} f(x)\,dx \overset{\text{set}}{=} .95$$

$$\Rightarrow \int_{.9\bar{x}}^{1.1\bar{x}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}} dx = .95$$

Let $x_1 = x - \bar{x}$; $dx_1 = dx$

$$= \int_{-.1\bar{x}}^{.1\bar{x}} \frac{1}{\sqrt{2\pi}\sigma} e^{-x_1^2/2\sigma^2} dx = .95 = 1 - 2Q\left(\frac{.1\bar{x}}{\sigma}\right)$$

$$\Rightarrow Q\left(\frac{.1\bar{x}}{\sigma}\right) = \frac{1.-.95}{2} = .025 \overset{\text{A-10}}{\Rightarrow} \frac{.1\bar{x}}{\sigma} = 1.96$$

$$\Rightarrow \sigma = \frac{.1\bar{x}}{1.96} = \frac{(.1)(1000)}{1.96} = \underline{\underline{51.0 \text{ ohms} = \sigma}}$$

B-30 (a.) $P(x \le 1) = F(1) = Q\left(\frac{5-1}{.6}\right) = Q(6.66)$

$$= \frac{1}{\sqrt{2\pi}(6.66)} e^{-(6.66)^2/2} = \underline{\underline{1.337 \times 10^{-11} = P(x \le 1)}}$$

(b.) $P(x \le 6) = F(6) = Q\left(\frac{5-6}{.6}\right) = Q(-1.667)$

$$= 1 - Q(1.667) \underset{\text{A-10}}{=} 1 - .04798 = \underline{\underline{.9520 = P(x \le 6)}}$$

B-34

$$f(y) = \left.\frac{f(x)}{\left|\frac{dy}{dx}\right|}\right|_{x_i = h^{-1}(y)} = \left.\frac{f(x_1)}{|2x_1|}\right|_{x_1=-\sqrt{y}} + \left.\frac{f(x_2)}{|2x_2|}\right|_{x_2=\sqrt{y}} ;\ y \geq 0$$

$y = x^2 \Rightarrow dy = 2x\,dx$

$$= \frac{f(-\sqrt{y})}{2\sqrt{y}} + \frac{f(+\sqrt{y})}{2\sqrt{y}} = \frac{1}{2\sqrt{y}}\left[\frac{1}{\sqrt{2\pi}\sigma}\left(e^{-(-\sqrt{y}-m)^2/2\sigma^2} + e^{-(\sqrt{y}-m)^2/2\sigma^2}\right)\right]$$

$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-m)^2/2\sigma^2}$

$$\Rightarrow f(y) = \begin{cases} \frac{1}{2\sigma\sqrt{2\pi y}}\left[e^{-\frac{(\sqrt{y}+m)^2}{2\sigma^2}} + e^{-\frac{(\sqrt{y}-m)^2}{2\sigma^2}}\right], & y \geq 0 \\ 0, & y < 0 \end{cases}$$

B-38 The input is $x(t) = A \sin w_m t$ where $A = 8$. The output consists of a quantized sinusoid similar to that shown in Fig. 3-8b. The PDF of the output, $y(t)$, will consist of δ functions at the quantized values. Thus,

$$f(y) = \sum_{k=1}^{M} P_k\, \delta(y - y_k)$$

where $M = 8$, the step size is $\delta = \frac{2A}{M} = \frac{16}{8} = 2$, and the

B-38 Cont'd

quantized values are: $\underline{\underline{y_k = \frac{(2k-M-1)}{2}\delta}}$

$$P_k = \int_{y_k-\delta/2}^{y_k+\delta/2} f_x(x)\,dx = \int_{y_k-\delta/2}^{y_k+\delta/2} \frac{1}{\pi\sqrt{A^2-x^2}}\,dx = \int_{y_k-\delta/2}^{y_k+\delta/2} \frac{\frac{1}{A}}{\pi\sqrt{1-\left(\frac{x}{A}\right)^2}}\,dx$$

Using (B-67)

$$= \frac{1}{\pi}\sin^{-1}\left(\frac{x}{A}\right)\Bigg|_{y_k-\delta/2}^{y_k+\delta/2} = \frac{1}{\pi}\left[\sin^{-1}\left(\frac{y_k+\delta/2}{A}\right) - \sin^{-1}\left(\frac{y_k-\delta/2}{A}\right)\right]$$

Using (A-29)

or

$$P_k = \frac{1}{\pi}\left[\sin^{-1}\left(\frac{(2k-M-1+1)\delta}{2A}\right) - \sin^{-1}\left(\frac{(2k-M-1-1)\delta}{2A}\right)\right]$$

$$P_k = \frac{1}{\pi}\left[\sin^{-1}\left(\frac{(2k-M)\frac{2A}{M}}{2A}\right) - \sin^{-1}\left(\frac{(2k-M-2)\frac{2A}{M}}{2A}\right)\right] = \underline{\underline{\frac{1}{\pi}\left[\sin^{-1}\left(\frac{2k-M}{M}\right) - \sin^{-1}\left(\frac{2k-M-2}{M}\right)\right]}}$$

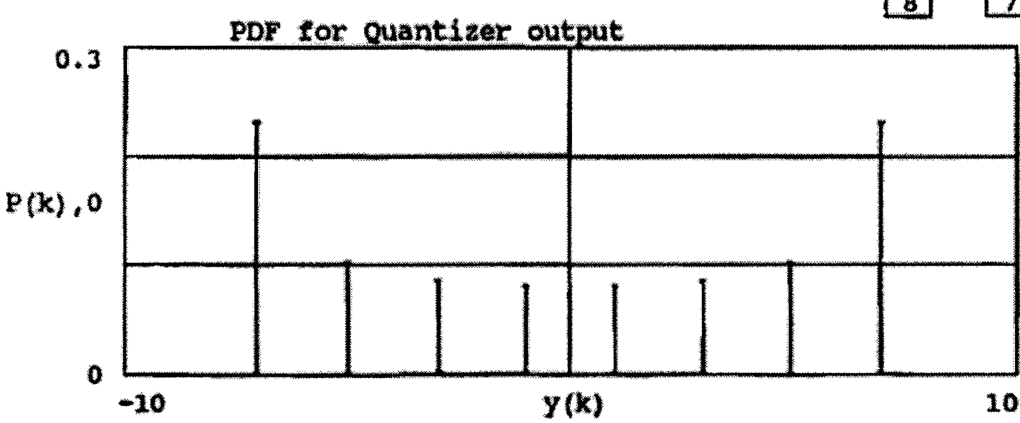
$A := 8 \qquad M := 8 \qquad \delta := 2\frac{A}{M} \qquad \delta = 2$ <--Step size

$k := 1..M$

$y(k) := (2k - M - 1)\, 0.5\cdot\delta$

$$P(k) := \frac{1}{\pi}\left[\operatorname{asin}\left[\frac{2\cdot k - M}{M}\right] - \operatorname{asin}\left[\frac{2k - M - 2}{M}\right]\right]$$

k	y(k)
1	-7
2	-5
3	-3
4	-1
5	1
6	3
7	5
8	7



B-40

$$\bar{y} = \int_{-\infty}^{\infty} y f(y)\,dy = \int_{0}^{\infty} \frac{y}{\sqrt{2\pi}\,\beta\Delta} e^{-y^2/2\beta^2\Delta^2} dy + \frac{1}{2}\int_{-\infty}^{\infty} \cancel{y\,\delta(y)\,dy}^{\,0}$$

$$= \left(\frac{-\beta\Delta}{\sqrt{2\pi}}\right)\int_{0}^{\infty} e^{-y^2/2\beta^2\Delta^2}\left(\frac{-y}{\beta^2\Delta^2}\right) dy$$

$$= \left(\frac{-\beta\Delta}{\sqrt{2\pi}}\right)\int_{0}^{-\infty} e^{z}\,dz = \left(\frac{-\beta\Delta}{\sqrt{2\pi}}\right) e^{z}\Big|_{0}^{-\infty}$$

$$\text{let } z = \frac{-y^2}{2\beta^2\Delta^2}$$

$$dz = \frac{-y}{\beta^2\Delta^2} dy$$

$$= \frac{\beta\Delta}{\sqrt{2\pi}} = \bar{y}$$

B-42 (a.) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x_1, x_2)\,dx_1\,dx_2 \overset{\text{set}}{=} 1$

$$\int_{x_2=0}^{4}\int_{x_1=0}^{1} k\,(x_1 + x_1 x_2)\,dx_1\,dx_2 = \int_{x_2=0}^{4}\left[\int_{x_1=0}^{1} k x_1 (1+x_2)\,dx_1\right] dx_2$$

$$= \int_{x_2=0}^{4}\left[k\,(1+x^2)\,\frac{x_1^2}{2}\Big|_0^1\right] dx_2 = \int_{x_2=0}^{4} k\,(1+x_2)\,\frac{1}{2}\,dx_2$$

$$= \frac{k}{2}\left[x_2 + \frac{x_2^2}{2}\right]\Big|_0^4 = \frac{k}{2}\left[4+8\right] = 6k = 1$$

$$\Longrightarrow k = 1/6$$

B-42 Cont'd

(b.)

$$f(x_1) = \int_0^4 f(x_1, x_2)\,dx_2 = \frac{1}{6}\int_0^4 x_1(1+x_2)\,dx_2$$

$$= \frac{1}{6}x_1\left[x_2 + x_2^2/2\right]\Big|_0^4 = \frac{1}{6}x_1[4+8] = \underline{\underline{2x_1}}$$

$$f(x_2) = \frac{1}{6}\int_0^1 (1+x_2)\,x_1\,dx_1 = \frac{(1+x_2)}{6}\frac{x_1^2}{2}\Big|_0^1$$

$$= \underline{\underline{\left(\frac{1+x_2}{12}\right)}}$$

$$f(x_1)f(x_2) = 2x_1\left(\frac{(1+x_2)}{12}\right) = \frac{1}{6}(x_1 + x_1x_2)$$

$$= f(x_1, x_2)$$

$\Longrightarrow$ x_1 & x_2 $\underline{\underline{\text{indep.}}}$

$$F_{x_1,x_2}(0.5, 2) = \frac{1}{6}\int_0^{.5} x_1 \int_0^2 (1+x_2)\,dx_2\,dx_1$$

$$= \frac{1}{6}\int_0^{.5} x_1\left[x_2 + \frac{x_2^2}{2}\right]\Big|_0^2 dx_1$$

$$= \frac{1}{6}\int_0^{.5} x_1(4)\,dx_1 = \frac{2}{3}\frac{x_1^2}{2}\Big|_0^{.5} = \underline{\underline{\frac{1}{12}}}$$

(d.)

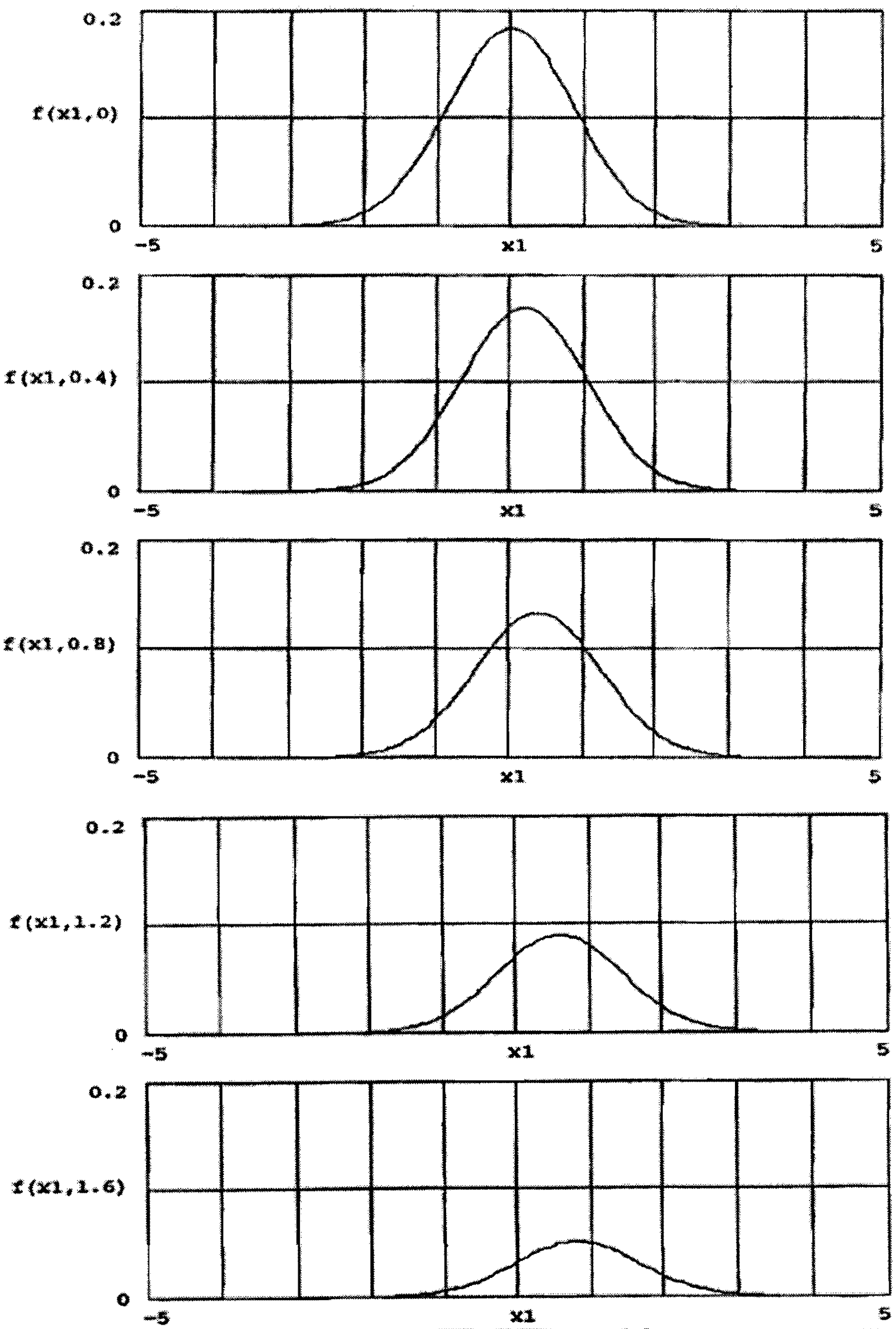
$$f_{x_2/x_1}(x_2/x_1) = \frac{f(x_2, x_1)}{f(x_1)} = \frac{f(x_2)f(x_1)}{f(x_1)}$$

$$= f(x_2) = \underline{\underline{\begin{cases} \frac{1}{12}(1+x_2); & 0 \le x_1 \le 1,\ 0 \le x_2 \le 4 \\ 0 & x_1 \text{ + } x_2 \text{ elsewhere} \end{cases}}}$$

B-45

$\sigma := 1 \qquad \rho := 0.5 \qquad x1 := -5, -4.95 \,..\, 5$

$$f(x1,x2) := \frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}}\, e^{-1\cdot\frac{x1^2 - 2\cdot\rho\; x1\; x2 + x2^2}{2\,\sigma^2\cdot\left[1-\rho^2\right]}}$$



B-52

$$mx := \begin{bmatrix} 2 \\ -1 \end{bmatrix} \qquad Cx := \begin{bmatrix} 5 & \frac{-2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & 4 \end{bmatrix} \qquad T := \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$$

(a.) Compute the mean vector for y:

$$my := T\ mx \qquad my = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix}$$

(b.) Compute the covariance matrix, Cy:

$$Cy := T \cdot Cx \cdot T^T \qquad Cy = \begin{bmatrix} 5.106 & 3.382 \\ 3.382 & 4.356 \end{bmatrix}$$

(c.) Compute the correlation coefficient for y1 and y2:

$$\rho := \frac{Cy_{0,1}}{\left[\sqrt{Cy_{0,0}}\right]\left[\sqrt{Cy_{1,1}}\right]} \qquad \rho = 0.717$$

B-54 Let $y_1 = A x_1 x_2$; $y_2 = x_2$

$$f(y_1, y_2) = \frac{f(x_1, x_2)}{|J(y/x)|} = \frac{f(y_1/Ax_2, y_2)}{|A y_2|}$$

$$J(y/x) = \det \begin{bmatrix} Ax_2 & Ax_1 \\ 0 & 1 \end{bmatrix} = Ax_2 = Ay_2$$

$$\Rightarrow f(y_1) = \int_{-\infty}^{\infty} \frac{f(y_1/Ay_2, y_2)}{|Ay_2|}\, dy_2$$

(a.) $$f(y) = \int_{-\infty}^{\infty} \frac{f(y/Ax_2, x_2)}{|Ax_2|}\, dx_2$$

(b.) $$f(y) = \int_{-\infty}^{\infty} \frac{f_{x_1}(y/Ax_2)\, f_{x_2}(x_2)}{|Ax_2|}\, dx_2$$

PEARSON
Prentice Hall

Upper Saddle River, New Jersey 07458