Chapter 4

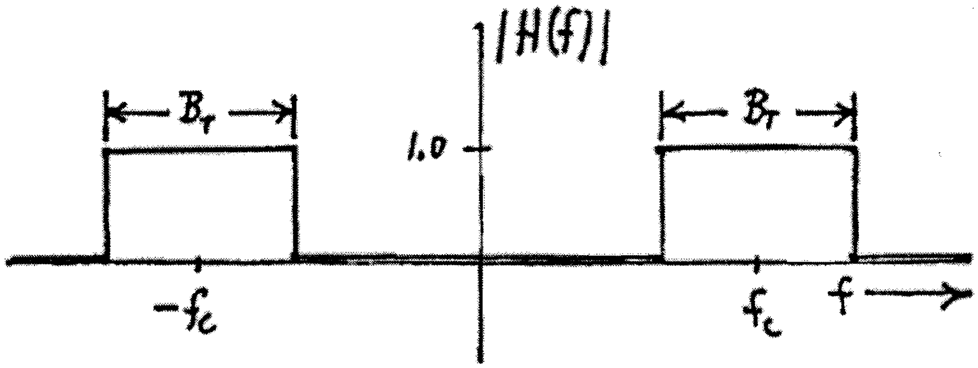
4-3 Using (2-26) with the help of Sec. A-5,

$$G(f) = A_c M(f) = 50[-j\delta(f-1000) + j\delta(f+1000)]$$

Substituting this into (4-15) and using $\delta(-f) = \delta(f)$, the voltage spectrum of this DSB-SC signal is

$$S(f) = -j25\,\delta(f-f_c-1000) + j25\,\delta(f-f_c+1000)$$
$$-j25\,\delta(f+f_c-1000) + j25\,\delta(f+f_c+1000)$$

4-9



4-9 Cont'd

(b) $v_2(t) = \text{Re}\{g_2(t)e^{j\omega_c t}\}$

where $g_2(t) = \frac{1}{2}g_1(t) * k(t) \leftrightarrow G_2(f) = \frac{1}{2}G_1(f)K(f)$

and $k(t) = \mathcal{F}^{-1}[K(f)]$

Also, $H(f) = \frac{1}{2}\left[K(f-f_c) + K^*(-f-f_c)\right]$

$$\Rightarrow \quad K(f) = \begin{Bmatrix} 2, & |f| < B_T/2 \\ 0, & f \text{ elsewhere} \end{Bmatrix}$$

Evaluate $k(t)$:

$$k(t) = \int_{-\frac{B_T}{2}}^{\frac{B_T}{2}} 2e^{j2\pi ft}\,df = 2B_T \frac{\sin(\pi B_T t)}{(\pi B_T t)}$$

Know that $g_1(t) = A\,\Pi\left(\frac{t}{T}\right) = A\begin{Bmatrix} 1, & |t| < T/2 \\ 0, & t \text{ elsewhere} \end{Bmatrix}$

$$\Rightarrow g_2(t) = \frac{1}{2}g_1(t) * k(t) = \frac{1}{\cancel{2}}\int_{-\frac{T}{2}}^{\frac{T}{2}} A\cancel{2}B_T \frac{\sin[\pi B_T(t-\lambda)]}{[\pi B_T(t-\lambda)]}\,d\lambda$$

Let $\lambda_1 = \pi B_T(t-\lambda) \Rightarrow d\lambda_1 = -\pi B_T\,d\lambda$

$$\Rightarrow g_2(t) = A\cancel{B_T}\int_{\pi B_T(t+\frac{T}{2})}^{\pi B_T(t-\frac{T}{2})} \frac{\sin\lambda_1}{\lambda_1}\left(-\frac{1}{\pi \cancel{B_T}}d\lambda_1\right)$$

$$= \frac{A}{\pi}\left[-\int_{\pi B_T(t+\frac{T}{2})}^{0}\frac{\sin\lambda_1}{\lambda_1}\,d\lambda_1 - \int_{0}^{\pi B_T(t-\frac{T}{2})}\frac{\sin\lambda_1}{\lambda_1}\,d\lambda_1\right]$$

$$\Rightarrow g_2(t) = \frac{A}{\pi}\left\{+\,\mathrm{Si}\left[\pi B_T\left(t+\tfrac{T}{2}\right)\right] - \mathrm{Si}\left[\pi B_T\left(t-\tfrac{T}{2}\right)\right]\right\}$$

and $v_2(t) = \text{Re}\{g_2(t)e^{j\omega_c t}\}$

4-9 Cont'd (b.)

Thus,

$$v_2(t) = \frac{A}{\pi}\left\{ Si\left[\pi B_T\left(t+\frac{T}{2}\right)\right] - Si\left[\pi B_T\left(t-\frac{T}{2}\right)\right]\right\} \cos(\omega_c t)$$

(c.) When $B_T = \frac{4}{T}$

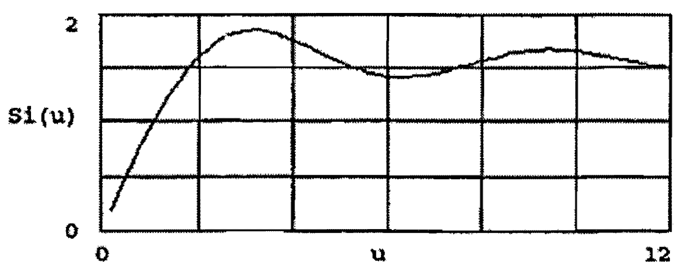
$$v_2(t) = \frac{A}{\pi}\left\{ Si\left[2\pi\left(\frac{2t}{T}+1\right)\right] - Si\left[2\pi\left(\frac{2t}{T}-1\right)\right]\right\} \cos(\omega_c t)$$

This is plotted with the help of the Si(u) function. (See p. 232 of Abramowitz and Stegun for a description of the Si(u) function.)

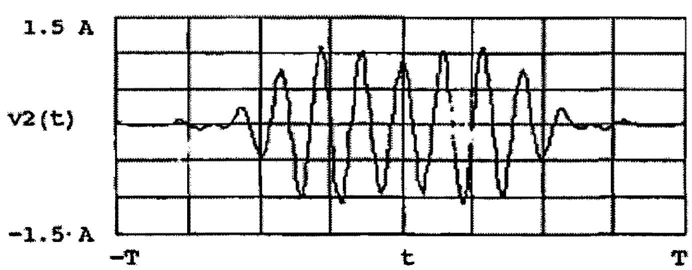
Using MathCAD we get:

A := 1 T := 1 ω := 2 π· 7 u := 0,0.2 ..12

$$Si(u) := \int_{0.001}^{u} \frac{\sin(x)}{x}\,dx \qquad t := -T, -T + 0.01 \,..T$$



$$v2(t) := \left[\frac{A}{\pi}\right]\cdot\left[Si\left[2\,\pi\left[2\cdot\frac{t}{T}+1\right]\right] - Si\left[2\cdot\pi\cdot\left[2\cdot\frac{t}{T}-1\right]\right]\right]\cos(\omega\, t)$$



4-11

$R := 400 \qquad L := 1.583\ 10^{-3} \qquad C := 1 \cdot 10^{-6}$

(a)

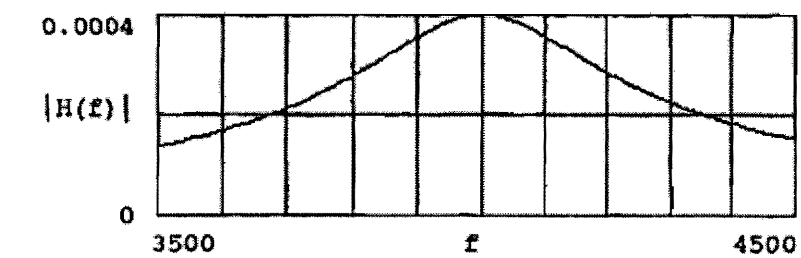
$$f0 := \frac{1}{2 \cdot \pi \sqrt{L \cdot C}} \qquad f0 = 4 \cdot 10^{3} \qquad Q := R \sqrt{\frac{C}{L}} \qquad Q = 10.054$$

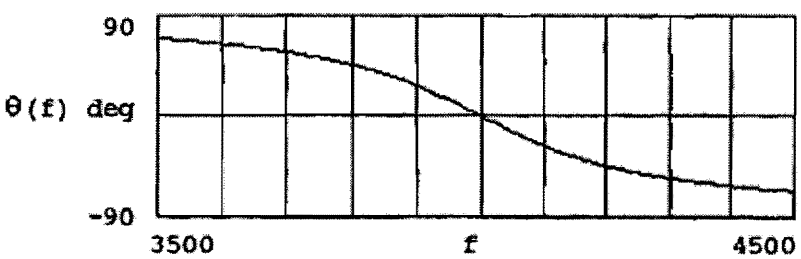
$$B := \frac{f0}{Q} \qquad B = 397.887$$

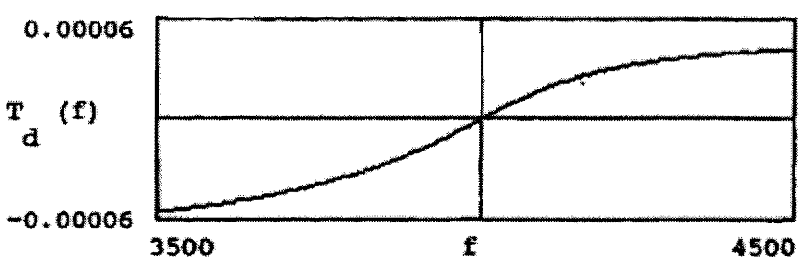
(b) $f := 3500, 3504\ ..4500 \qquad rad \equiv 1 \qquad deg \equiv \frac{rad}{\pi} \cdot 180$

$$H(f) := \frac{j\ \ 2 \cdot \pi\ f}{(j\ \ 2\ \pi\ f)^{2} + \left[2 \cdot \pi \cdot \frac{f0}{Q}\right] \cdot (j\ \ 2 \cdot \pi \cdot f) + (2 \cdot \pi \cdot f0)^{2}}$$

$$\theta(f) := \arg(H(f)) \qquad T_{d}(f) := \frac{1}{-2 \cdot \pi\ f} \cdot \theta(f)$$



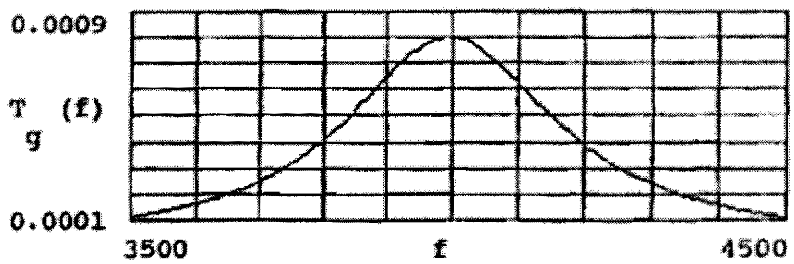




4-11
Cont'd

(c)

$$T_g(f) := \frac{1}{-2\cdot\pi}\,\frac{d}{df}\,(\theta(f)\ \mathrm{rad})$$



(d.) The group delay variation over the 200 Hz signal bandwidth (about 4 kHz) is about 0.2 msec. The period of a 200 Hz signal is 5 msec. Thus, the group delay variation is negligable and, consequently the distortion is negligable.

4-14

(a.) $s(t) = \mathrm{Re}\{500\, e^{j\omega_c t}\} + \mathrm{Re}\{-j100\, e^{j(\omega_c+\omega_a)t} + j100\, e^{j(\omega_c-\omega_a)t}\}$

$$s(t) = \mathrm{Re}\left\{500\left[1 - j(2j)\frac{100}{500}\left(\frac{e^{j\omega_a t} - e^{-j\omega_a t}}{2j}\right)\right] e^{j\omega_c t}\right\}$$

$$= \mathrm{Re}\left\{500\,[\underbrace{1 + \tfrac{2}{5}\sin(\omega_a t)}_{1+m(t)}]\, e^{j\omega_c t}\right\} \qquad AM$$

$$\Rightarrow\ \underline{\underline{g(t) = 500 + 200\sin(\omega_a t)}}\ ,\quad \underline{\underline{m(t) = 0.4\sin(\omega_a t)}}$$

(b.) $x(t) = \mathrm{Re}\{g(t)\} = \underline{\underline{500 + 200\sin(\omega_a t)}}\ ,\quad y(t) = \mathrm{Im}\{g(t)\} = \underline{\underline{0}}$

(c.) $R(t) = |g(t)| = \underline{\underline{500 + 200\sin(\omega_a t)}}\ ,\quad \theta(t) = \angle g(t) = \underline{\underline{0^\circ}}$

(d.) $$P = \frac{\frac{1}{2}\langle |g(t)|^2\rangle}{50} = \frac{1}{100}\left[(500)^2 + 2\times10^5 \overset{0}{\cancel{\langle\sin\omega_a t\rangle}} + (200)^2 \overset{\frac{1}{2}}{\cancel{\langle\sin^2(\omega_a t)\rangle}}\right]$$

$$\Rightarrow\ \underline{\underline{P = 2{,}700\ \text{watts}}}$$

4-15

(a) $S(f) = 100\,\mathcal{F}[\sin(\omega_c+\omega_a)t] + 500\,\mathcal{F}[\cos \omega_c t] - 100\,\mathcal{F}[\sin(\omega_c-\omega_a)t]$

Aside: We know that

$$\mathcal{F}[\sin(\omega_x t)] = \frac{1}{2j}[\delta(f-f_x) - \delta(f+f_x)]$$

and $$\mathcal{F}[\cos(\omega_x t)] = \frac{1}{2}[\delta(f-f_x)+\delta(f+f_x)]$$

Thus

$$S(f) = -j50[\delta(f-(f_c+f_a)) - \delta(f+(f_c+f_a))] + 250[\delta(f-f_c)+\delta(f+f_c)] + j50[\delta(f-(f_c-f_a)) - \delta(f+(f_c-f_a))]$$

$$\Rightarrow S(f) = 250[\delta(f-f_c)+\delta(f+f_c)] + j50[\delta(f+f_c+f_a) - \delta(f-f_c-f_a) + \delta(f-f_c+f_a) - \delta(f+f_c-f_a)] \quad (1)$$

(b) Let $g(t) = 500 + 200\sin(\omega_a t)$

Thus, it can be shown that $s(t) = \text{Re}[g(t)e^{j\omega_c t}] = s(t)$ of 4-9

$$G(f) = \mathcal{F}[g(t)] = 500\,\delta(f) - j100[\delta(f-f_a) - \delta(f+f_a)]$$

Using $S(f) = \frac{1}{2}[G(f-f_c) + G^*(-f-f_c)]$ & $\delta(-f) = \delta(f)$

$$\Rightarrow S(f) = 250[\delta(f-f_c) + \cancel{\delta(-f-f_c)}] \quad \delta(f+f_c)$$
$$-j50[\delta(f-f_c-f_a) - \cancel{\delta(-f-f_c-f_a)}] \quad \delta(f+f_c+f_a)$$
$$+j50[\delta(f-f_c+f_a) - \cancel{\delta(-f-f_c+f_a)}] \quad \delta(f+f_c-f_a)$$

$$\Rightarrow S(f) = 250[\delta(f-f_c)+\delta(f+f_c)] + j50[\delta(f-f_c+f_a) - \delta(f-f_c-f_a) + \delta(f+f_c+f_a) - \delta(f+f_c-f_a)] \quad (2)$$

Thus (1) = (2)

4-19

$B := 100 \qquad f := 10, 20 .. 1000 \qquad rad \equiv 1 \qquad deg \equiv \frac{rad}{\pi} \cdot 180$

$$H1(f) := \frac{1}{\left[1 - \left[\frac{f}{B}\right]^2\right] + j \left[\sqrt{2}\, \frac{f}{B}\right]}$$

$\theta 1(f) := \arg(H1(f))$

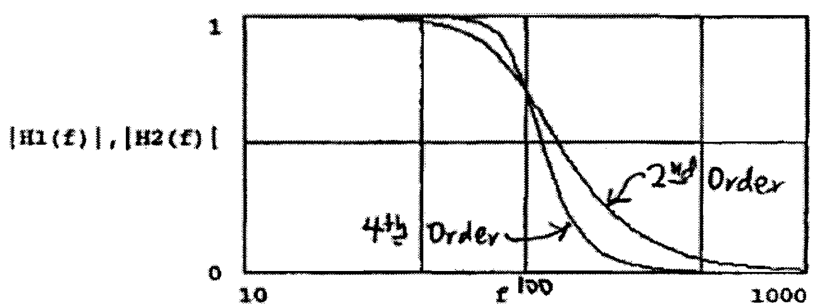
$$Td1(f) := \frac{1}{-2 \cdot \pi \cdot f} \cdot \theta 1(f)$$

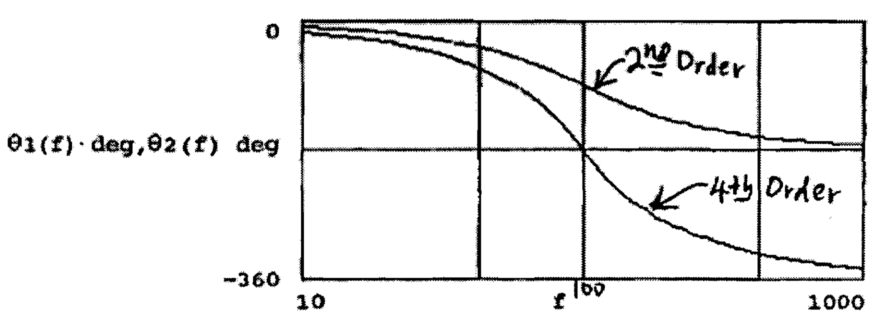
$$H2(f) := \frac{1}{\left[1 - \left[\frac{f}{B}\right]^2 + 0.765 j \cdot \frac{f}{B}\right]} \cdot \frac{1}{\left[1 - \left[\frac{f}{B}\right]^2 + 1.848 j \cdot \frac{f}{B}\right]}$$

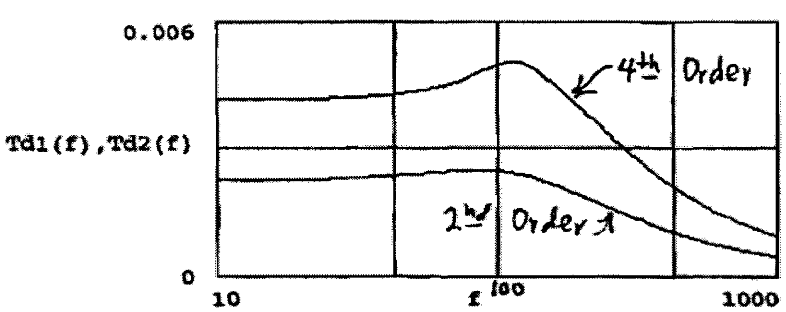
$\theta(f) := \arg(H2(f))$

$\theta 2(f) := \mathrm{if}(f < 100, \theta(f), \theta(f) - 2 \cdot \pi)$

$$Td2(f) := \frac{1}{-2 \cdot \pi\, f} \cdot \theta 2(f)$$







4-22

$t := 0, 0.1 .. 4$ $\qquad m := 1 .. 6$

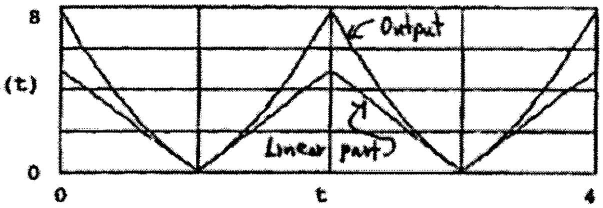
(a.)

$$\mathrm{vin}(t) := \frac{1}{2} + \frac{4}{\pi^2}\left[\sum_m \left[\frac{\cos((2\cdot m - 1)\cdot \pi \cdot t)}{(2\,m-1)^2}\right]\right]$$

Note: vin(t) is the Fourier series for a triangle waveform.

$$\mathrm{vout}(t) := 5\cdot \mathrm{vin}(t) + 1.5\cdot(\mathrm{vin}(t))^2 + 1.5\cdot(\mathrm{vin}(t))^3$$

$\mathrm{vol}(t) := 5\cdot \mathrm{vin}(t)$ $\qquad$ <-----Linear part of the output.



(b.)

$M := 5 \qquad N := 2^M \qquad N = 32 \qquad k := 0 .. N-1 \qquad T := 2$

$dt := \frac{T}{N} \qquad dt = 0.063 \qquad t_k := k\cdot dt$

$vo_k := \mathrm{vout}(k\cdot dt) \qquad vol_k := \mathrm{vol}(k\,dt) \qquad n := 0 .. N-1$

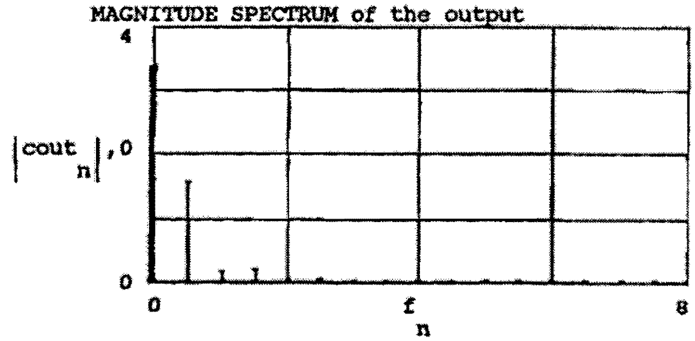
Since the signal is periodic, the spectrum will consist of delta functions (which can't be plotted directly since the delta function has an infinite value). However the weights of the delta functions are finite and can be plotted. The weights may be obtained from the complex Fourier series coefficients. Furthermore, the complex Fourier series coefficients may be calculated using the FFT by substituting (2-178) into (2-186). Thus,

$$cout := \left[\frac{1}{\sqrt{N}}\right]\cdot \mathrm{icfft}(vo) \qquad col := \left[\frac{1}{\sqrt{N}}\right] \mathrm{icfft}(vol)$$

$f_n := \frac{n}{T} \qquad f_1 = 0.5 \qquad fs := \frac{1}{dt} \qquad fs = 16$

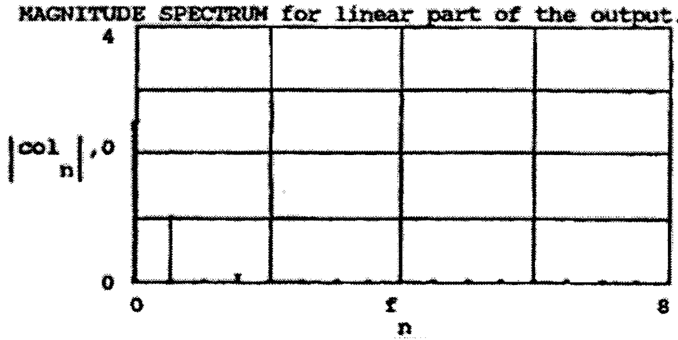
MAGNITUDE SPECTRUM of the output

$cout_0 = 3.375$

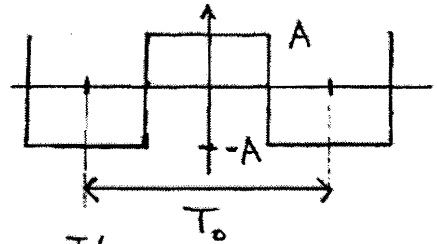


MAGNITUDE SPECTRUM for linear part of the output.

$col_0 = 2.5$



4-25 The output is a square wave as shown

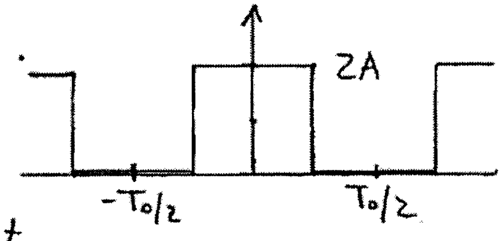


$$v(t) = \sum_{n=0}^{\infty} V_n \cos(n\omega_0 t)$$

where, using (2-96),

$$b_n = 0 \quad \text{and} \quad a_n = V_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} v(t)\cos(n\omega_0 t)\,dt$$

Since we are only interested in V_n for $n \geq 1$, we can shift the DC level of the waveform to make the integral easier to evaluate and yet the V_n will remain the same for $n \neq 0$.



$$V_n = \frac{2}{T_0}\int_{-T_0/4}^{T_0/4} 2A\cos(n\omega_0 t)\,dt$$

$$= \frac{2}{T_0}(2A)\left.\frac{\sin(n\omega_0 t)}{n\omega_0}\right|_{-T_0/4}^{T_0/4} = \frac{4A}{n2\pi}\, 2\sin\left(\frac{n\pi}{2}\right)$$

$$= \frac{4A}{n\pi}\begin{cases} 0, & n = \text{even} \\ +1, & n = 1, 5, 9, \ldots \\ -1, & n = 3, 7, 11, \ldots \end{cases}$$

$$\Rightarrow V_n^2 = \begin{cases} 0, & n \text{ even} \\ \left(\frac{4A}{n\pi}\right)^2, & n \text{ odd} \end{cases}$$

Using (4-47)

$$\text{THD \%} = \sqrt{\frac{\sum_{n=2}^{\infty} V_n^2}{V_1^2}} \times 100 = \sqrt{\frac{\sum_{n=3}^{\infty}\left(\frac{4A}{n\pi}\right)^2}{\left(4A/\pi\right)^2}}$$

4-25. Cont'd

$$THD\% = \sqrt{\sum_{\substack{n=3 \\ n=\text{odd}}}^{\infty} \frac{1}{n^2}} \times 100 = \underline{\underline{48.3\%}}$$

Using programable calculator

<u>Check</u>:

$$THD\% = \sqrt{\frac{\text{Total Power} - \left(\frac{V_1}{\sqrt{2}}\right)^2}{\left(\frac{V_1}{\sqrt{2}}\right)^2}} \times 100$$

$$= \sqrt{\frac{A^2 - \left(\frac{4A}{\pi\sqrt{2}}\right)^2}{\left(\frac{4A}{\pi\sqrt{2}}\right)^2}} \times 100$$

$$= \sqrt{\frac{\pi^2 - 8}{8}}\,(100) = \sqrt{0.2337}\,(100) = \underline{\underline{48.3\%}}$$

4-28 $s(t) = A_c[m(t)\cos w_c t \mp \hat{m}(t)\sin w_c t]$

$$= \text{Re}\{A_c[m(t) \pm j\hat{m}(t)]\,e^{jw_c t}\}$$

$$\Rightarrow g(t) = A_c[m(t) \pm j\,\hat{m}(t)] = R(t)\,\underline{/\theta(t)}$$

Output of Envelope Detector is

$$v_{out}(t) = KR(t) = k|g(t)|$$

$$\Rightarrow v_{out}(t) = kA_c\sqrt{m^2(t) + \hat{m}^2(t)} \neq k\,m(t)$$

The output is <u>distorted</u>.

4-33

$$\frac{d\theta_e(t)}{dt} = \frac{d\theta_i(t)}{dt} - K_d k_v\, \theta_e(t) * f(t)$$

$$\Rightarrow \quad s\,\Theta_e(s) = s\,\Theta_i(s) - k_d k_v\, \Theta_e(s) F(s)$$

or

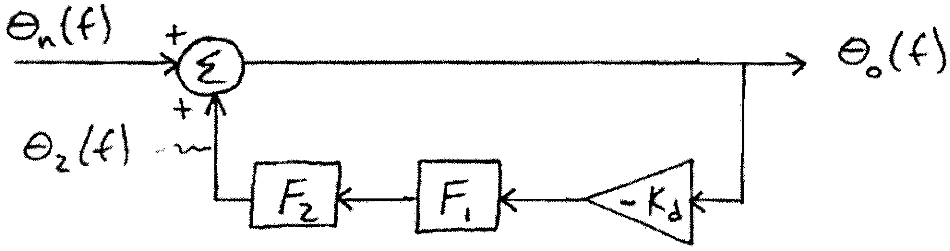
$$\Theta_e(s) = \frac{s\,\Theta_i(s)}{s + k_d k_v\, F(s)}$$

Final value theorem:

$$\lim_{t\to\infty} \theta_e(t) = \lim_{s\to 0}\left[s\,\Theta_e(s)\right] = \lim_{s\to 0} \frac{s^2\,\Theta_i(s)}{s + k_d k_v F(s)}$$

$$\Rightarrow \text{ If } F(0) \neq 0 \Rightarrow \underline{\underline{\lim_{t\to\infty} \theta_e(t) = 0}}$$

4-35 (a.)



$$\Theta_o(f) = \Theta_n(f) + \Theta_2(f)$$

where $\Theta_2(f) = \Theta_o(f)\left[-K_d\, F_1(f)\, F_2(f)\right]$

$$\Rightarrow \Theta_o(f) = \Theta_n(f) - K_d\, F_1(f)\, F_2(f)\, \Theta_o(f)$$

$$\Theta_o(f)\left[1 + K_d F_1 F_2\right] = \Theta_n(f)$$

$$\frac{\Theta_o(f)}{\Theta_n(f)} = \frac{1}{1 + K_d F_1 F_2} = \frac{1}{1 + \frac{K_d K_v F_1(f)}{j2\pi f}}$$

$F_2(f) = K_v / j2\pi f$

4-35 (a.) Cont'd

$$\frac{\Theta_o(f)}{\Theta_n(f)} = \frac{j2\pi f}{j2\pi f + K_d K_v F_1(f)}$$

(b.) $F_1(f) = \frac{1}{1 + jf/f_1}$; $f_1 = \frac{1}{2\pi RC}$

$$\frac{\Theta_o(f)}{\Theta_n(f)} = \frac{1}{1 + \frac{K_d K_v}{j2\pi f(1 + jf/f_1)}}$$

$$= \frac{j2\pi f(1 + jf/f_1)}{j2\pi f(1 + jf/f_1) + K_d K_v}$$

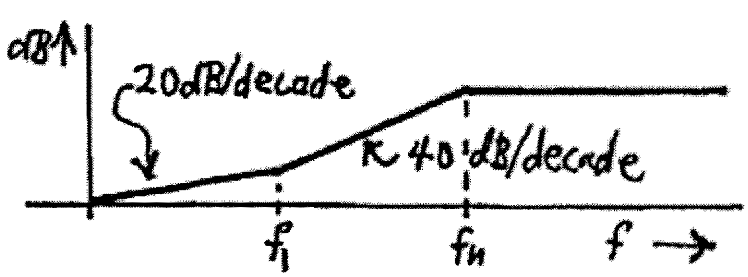
$$= \frac{1}{K_d K_v} \frac{j2\pi f(1 + jf/f_1)}{1 + j\frac{2\pi f}{K_d K_v} - \frac{2\pi f^2}{K_d K_v f_1}}$$

$$= \frac{1}{K_d K_v} \frac{j2\pi f(1 + jf/f_1)}{1 + j2\zeta\frac{f}{f_n} - \left(\frac{f}{f_n}\right)^2}$$

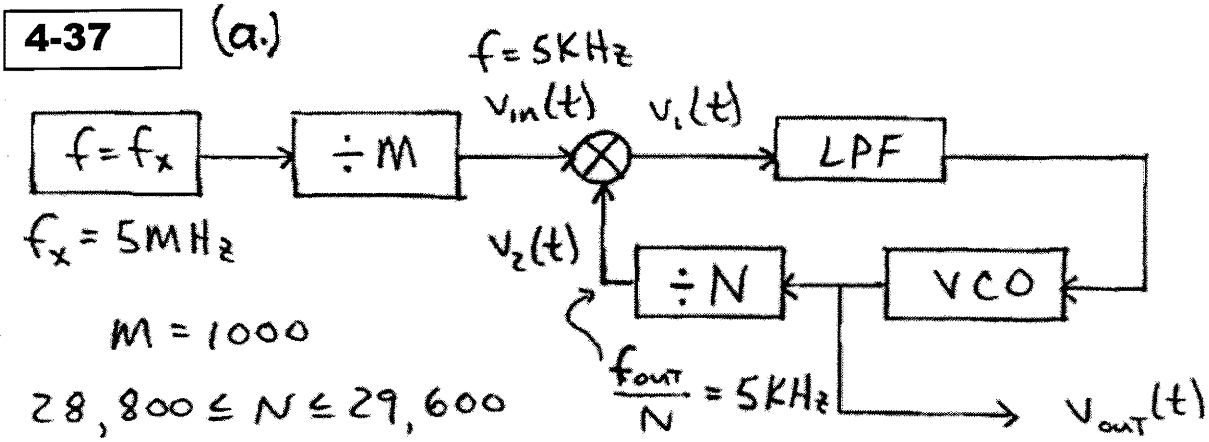
where $f_n = \sqrt{\frac{K_d K_v f_1}{2\pi}}$; $\zeta = \frac{f_1}{2f_n}$

$$0 \le \zeta \le 1 \implies f_n \ge \frac{f_1}{2}$$

$\left|\frac{\Theta_o(f)}{\Theta_n(f)}\right|_{dB}$



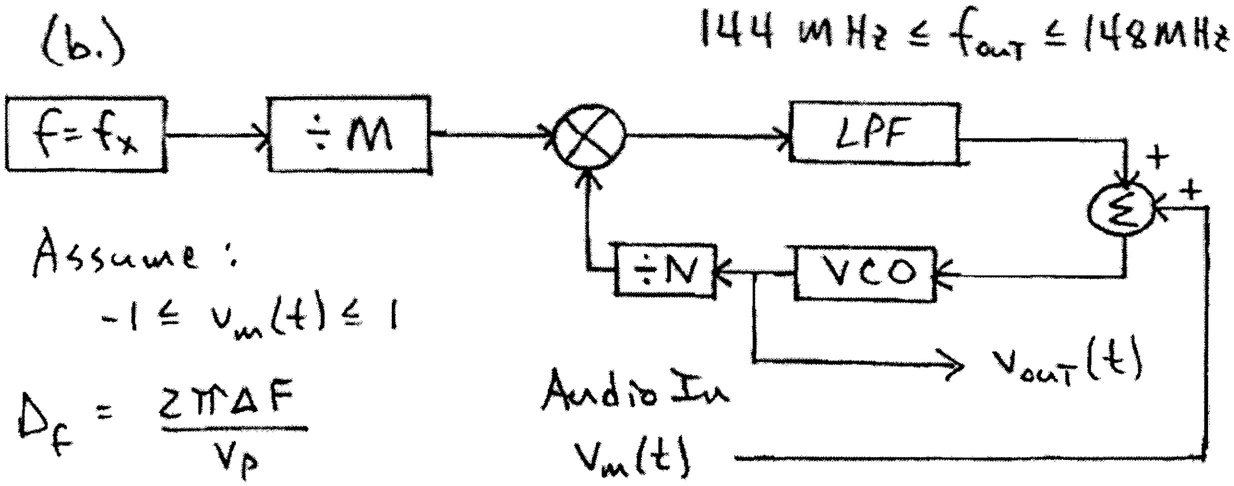
4-37 (a.)



$M = 1000$

$28,800 \le N \le 29,600$

(b.)



Assume:

$-1 \le v_m(t) \le 1$

$$D_f = \frac{2\pi \Delta F}{V_p}$$

$$= 2\pi(5000)\ \frac{\text{rad.}}{\text{volt} \cdot \text{sec}}$$

4-42 (a.) $f_{LO} = 96.9 + 10.7 = \underline{\underline{107.6 \text{ MHz}}}$

(b.) RF: Flat bandpass over 96.81 MHz to 96.99 MHz and reject image frequency of 118.3 MHz

IF: Flat bandpass over 10.61 MHz to 10.79 MHz and reject adjacent channel signals on each side of this bandpass

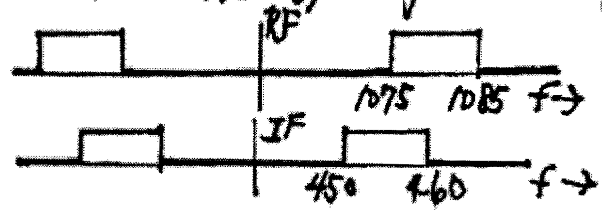
(c.) $f_{image} \doteq f_c + 2f_{if} = 96.9 + 2(10.7) = \underline{\underline{118.3 \text{ MHz}}}$

4-47

(a.) RF filter ⇒ $f_c \pm B/2$; IF filter ⇒ $f_{if} \pm B/2$ where $B = 10$ kHz be since the AM channel spacing is 10 kHz.

RF: 1080 ± 5 kHz

IF: 455 ± 5 kHz



(b.) $f_{image} = f_c + 2f_{if} = 1990$ kHz