# Chapter 2

**2-1**

$v(t) = A\sin\omega_0 t$ ; $V_{rms}^2 = \langle v^2(t)\rangle$

$$\langle v^2(t)\rangle = \frac{1}{T_0}\int_0^{T_0} A^2\sin^2\omega_0 t\,dt = \frac{A^2}{T_0}\int_0^{T_0}[1-\underbrace{\cos^2(\omega_0 t)}_{\frac{1}{2}[1+\cos(2\omega_0 t)]}]\,dt$$

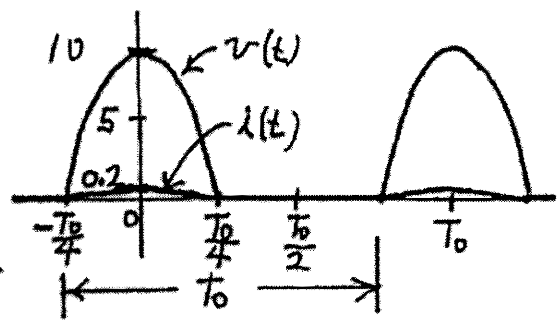
$$= \frac{A^2}{T_0}\left[T_0 - \frac{T_0}{2} - \frac{1}{2}\underbrace{\int_0^{T_0}\cos(2\omega_0 t)\,dt}_{0}\right] = \frac{A^2}{\cancel{T_0}}\left(\frac{\cancel{T_0}}{2}\right)$$

$$\Rightarrow V_{rms} = \sqrt{\langle v^2(t)\rangle} = \sqrt{\frac{A^2}{2}} = \underline{\underline{\frac{A}{\sqrt{2}}}}$$

**2-4**

(a.) $i(t) = \dfrac{v(t)}{R} = \dfrac{v(t)}{50}$

$$i(t) = \begin{cases} 0.2\cos(\omega_0 t), & |t - nT_0| < \frac{T_0}{2} \\ 0, & \text{elsewhere} \end{cases}$$




(b.)

$$V_{DC} = \langle v(t)\rangle = \frac{V_p}{T_0}\int_{-\frac{T_0}{4}}^{\frac{T_0}{4}}\cos(\omega_0 t)\,dt = \frac{2V_p}{T_0}\,\frac{\sin(\omega_0\frac{T_0}{4})}{\omega_0}$$

$$= \frac{2V_p}{\cancel{T_0}}\,\frac{\sin\left(\frac{2\pi}{\cancel{T_0}}\frac{\cancel{T_0}}{4}\right)}{\frac{2\pi}{\cancel{T_0}}} = \frac{\cancel{2}}{\cancel{2}\pi}V_p\sin\left(\frac{\pi}{2}\right)$$

$$\Rightarrow V_{DC} = \frac{V_p}{\pi} \underset{V_p=10}{=} \frac{10}{\pi} = \underline{\underline{3.183 \text{ volts}}}$$

$$\Rightarrow I_{DC} = \frac{I_p}{\pi} \underset{I_p=0.2}{=} \frac{0.2}{\pi} = \underline{\underline{0.064 \text{ Amps}}}$$

2-4. Cont'd (c.) $V_{rms}^2 = \langle v^2(t) \rangle = \frac{1}{T_0}\int_0^{T_0/2} v^2(t)dt = \frac{V_p^2}{T_0}\int_{-T_0/4}^{T_0/4} \cos^2\omega_0 t\, dt$

$$\Rightarrow V_{rms}^2 = \frac{V_p^2}{T_0}\int_{-T_0/4}^{T_0/4}\frac{1}{2}[1+\cos(2\omega_0 t)]dt = \frac{V_p^2}{2T_0}\left[2\frac{T_0}{4} + \frac{\sin(2\omega_0 t)}{2\omega_0}\Big|_{-T_0/4}^{T_0/4}\right]$$

$$= \frac{V_p^2}{2T_0}\frac{2T_0}{4} = \frac{V_p^2}{4} = V_{rms}^2$$

$$\Rightarrow V_{rms} = \frac{V_p}{2} = \frac{10}{2} = \underline{\underline{5 \text{ volts rms}}}$$

$$I_{rms} = \frac{I_p}{2} = \frac{0.2}{2} = \underline{\underline{0.1 \text{ amps}}}$$

(d) $P = \langle p(t) \rangle = V_{rms} I_{rms} = (5)(0.1) = \underline{\underline{0.5 \text{ watts}}}$

**2-10** $P_{in} = I_{rms}^2 R_{in} = (0.5\times10^{-3})^2(2\times10^3) = 5.0\times10^{-4}\,W$

$$P_{out} = \frac{V_{rms}^2}{R_{Load}} = \frac{100}{50} = 2\,W$$

$$dB = 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right) = 10\log\left(\frac{2}{5.0\times10^{-4}}\right) = \underline{\underline{36\ dB}}$$

**2-11** (a.) $P_{in} = \frac{V_{rms}^2}{R_{in}} = \frac{(3.5\times10^{-6})^2}{300} = \underline{\underline{4.083\times10^{-14}\,W}}$

(b.) $dBm = 10\log_{10}\left(\frac{P}{10^{-3}}\right) = 10\log_{10}\left(\frac{4.08\times10^{-14}}{10^{-3}}\right) = \underline{\underline{-103.9\,dBm}}$

(c.) $P_{in} = \frac{V_{rms}^2}{75} = 4.08\times10^{-14}$

$$\Rightarrow V_{rms} = \sqrt{75(4.08\times10^{-14})} = \underline{\underline{1.75\,\mu\text{volts}}}$$

**2-15**

$$W(f) = \int_{-\infty}^{\infty} w(t) e^{-j\omega t} dt = \int_1^{\infty} e^{-(\alpha + j\omega)t} dt$$

$$= \frac{e^{-(\alpha+j\omega)t}}{-(\alpha + j\omega)}\Bigg|_1^{\infty} = \underline{\underline{\frac{e^{-\alpha} e^{-j2\pi f}}{\alpha + j2\pi f} = W(f)}}$$

**2-18**

$$S(f) = \int_{-\infty}^{\infty} s(t) e^{-j\omega t} dt = \int_0^{T_0} A t e^{-j\omega t} dt$$

$$= A\left[e^{-j\omega t}\left(\frac{t}{-j\omega} + \frac{1}{\omega^2}\right)\right]\Bigg|_0^{T_0}$$

$$\int x e^{ax} dx = e^{ax}\left[\frac{x}{a} - \frac{1}{a^2}\right]$$

$$= A\left\{e^{-j\omega T_0}\left(\frac{T_0}{-j\omega} + \frac{1}{\omega^2}\right) - \frac{1}{\omega^2}\right\}$$

$$= \frac{A}{(2\pi f)^2}\left\{e^{-j2\pi f T_0} - 1\right\} + \frac{A T_0 e^{-j2\pi f T_0}}{-j2\pi f}$$

$$\Rightarrow \underline{\underline{S(f) = \frac{-A}{(2\pi f)^2} + A e^{-j2\pi f T_0}\left(\frac{1}{(2\pi f)^2} + j\frac{T_0}{2\pi f}\right)}}$$

## 2-24

(a.)

$M := 8 \qquad N := 2^M \qquad N = 256 \qquad k := 0 \,..\, N-1 \qquad T := 40$

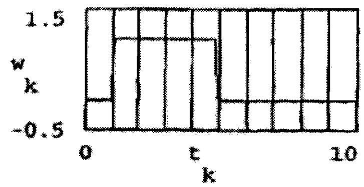
$dt := \frac{T}{N} \qquad t_k := k\,dt - 10$

$w_k := \Phi\left[t_k - 1.0\right] - \Phi\left[t_k - 5.0\right]$

$w_0 = 0 \qquad dt = 0.156$

$f4 := 4$

WAVEFORM




$n := 0 \,..\, N-1$

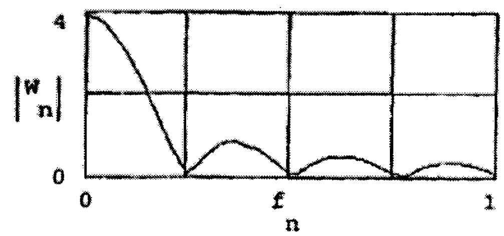
$W := dt\left[\sqrt{N}\right]\ \mathrm{icfft}(w)$

$f_n := \frac{n}{T} \qquad fs := \frac{1}{dt}$

$W_0 = 3.906 \qquad fs = 6.4$

$f_1 = 0.025 \qquad f4 = 4$

MAGNITUDE SPECTRUM out to 4th null

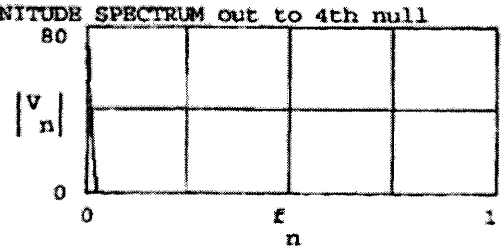



(b.) $v_k := 2.0$

$V := dt\left[\sqrt{N}\right]\ \mathrm{icfft}(v)$

$V_0 = 80$

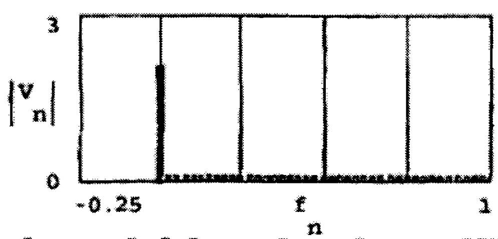
MAGNITUDE SPECTRUM out to 4th null




NOTE: The FFT cannot give the correct amplitude value for a delta function since the delta function has an infinite amplitude. However the area under the FFT result that approximates the delta function will be approximately the correct weight for the delta function. The value for the weight of the delta function may be calculated directily via the FFT by using (2-187). This is shown below.

$V := \left[\frac{1}{\sqrt{N}}\right]\ \mathrm{icfft}(v)$

$V_0 = 2$ <--Weight of $\delta$

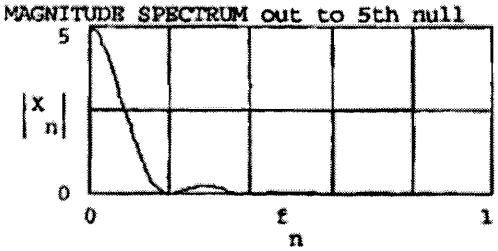


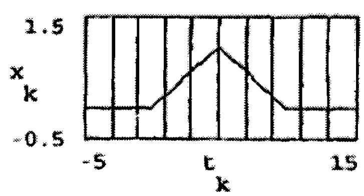

(c.)

$x_k := 0.2\left[t_k\left[\Phi\left[t_k\right] - \Phi\left[t_k - 5\right]\right] - \left[t_k - 10\right]\left[\Phi\left[t_k - 5\right] - \Phi\left[t_k - 10\right]\right]\right]$

$X := dt\left[\sqrt{N}\right]\ \mathrm{icfft}(x)$

MAGNITUDE SPECTRUM out to 5th null

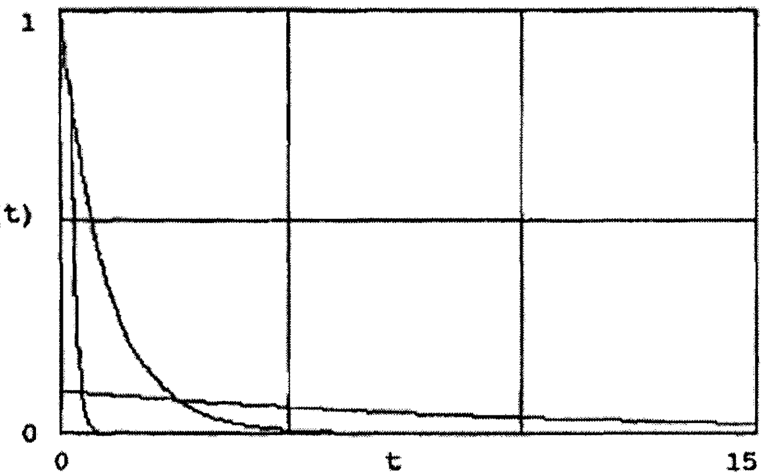



$X_0 = 5$

**2-32** (a) $t := 0, 0.05 .. 15$

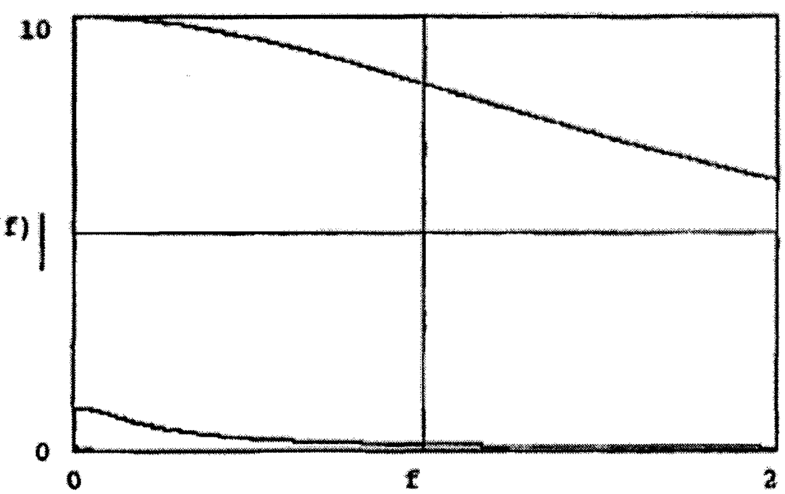
$v_1(t) := 0.1 \cdot e^{-0.1\,t}$ $\quad v_2(t) := e^{-t}$ $\quad v_3(t) := 10 \cdot e^{-10 \cdot t}$




(b) $f := 0, 0.001 .. 2$

$V_1(f) := \dfrac{0.1}{1 + j \cdot 20\,\pi \cdot f}$ $\quad V_2(f) := \dfrac{1}{1 + j\ 2 \cdot \pi \cdot f}$ $\quad V_3(f) := \dfrac{10}{1 + j \cdot 0.2 \cdot \pi \cdot f}$

$V_1(0) = 0.1$ $\quad V_2(0) = 1$ $\quad V_3(0) = 10$



**2-37**

$$w(t) = w_1(t)\, w_2(t)$$

$$W(f) = \int_{-\infty}^{\infty} w(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} w_1(t)\, w_2(t)\, e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} W_1(\lambda) e^{j2\pi\lambda t} d\lambda \right] w_2(t)\, e^{-j2\pi f t} dt$$

$$= \int_{-\infty}^{\infty} W_1(\lambda) \underbrace{\int_{-\infty}^{\infty} w_2(t)\, e^{-j2\pi(f-\lambda)t} dt}_{W_2(f-\lambda)}\, d\lambda = \int_{-\infty}^{\infty} W_1(\lambda) W_2(f-\lambda) d\lambda = W(f)$$

**2-42**

(a.) $$\int_{-\infty}^{\infty} \frac{\sin 4\lambda}{4\lambda} \delta(t-\lambda) d\lambda = \underline{\underline{\frac{\sin(4t)}{4t}}}$$

(b.) $$\int_{-\infty}^{\infty} (\lambda^3 - 1)\, \delta(2-\lambda)\, d\lambda = 2^3 - 1 = \underline{\underline{7}}$$

**2-48** (a.) $$V_{DC} = \frac{1}{T} \int_0^T s(t)\, dt$$

$$= \frac{1}{3} \left[ -2A + \int_2^3 A \sin(\pi(t-2))\, dt \right]$$

let $t_1 = t-2$

$$= \frac{1}{3} \left[ -2A + A \int_0^1 \sin \pi t_1\, dt_1 \right]$$

$$= \frac{A}{3} \left[ -2 - \frac{\cos \pi t_1}{\pi} \Big|_0^1 \right] = \frac{A}{3} \left[ -2 + \frac{2}{\pi} \right]$$

$$= -\frac{2A}{3\pi} [\pi - 1] = \underline{\underline{-0.454 A = V_{DC}}}$$

2-48 Cont'd

(b.) $V_{rms}^2 = \frac{1}{T}\int_0^T s^2(t)\,dt$

$$= \frac{1}{3}\left[(-A)^2 2 + \int_0^1 [A\sin \pi t_1]^2 dt_1\right]$$

$$= \frac{A^2}{3}\left[2 + \int_0^1 \sin^2(\pi t_1)\,dt_1\right]$$

$$= \frac{A^2}{3}\left[2 + \frac{1}{2}\int_0^1 (1-\cos 2\pi t_1)\,dt_1\right]$$

$$= \frac{A^2}{3}\left[2 + \frac{1}{2} - \frac{1}{2}\underbrace{\cancel{\frac{\sin 2\pi t_1}{2\pi}\Big|_0^1}}_{0}\right] = \frac{A^2}{3}\left(\frac{5}{2}\right)$$

$$V_{rms} = \sqrt{\frac{5}{6}}\,A = \underline{\underline{0.913\,A = V_{rms}}}$$

(c.) $s(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$ ; $\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{3}$

$$c_n = \frac{1}{T}\int_0^{T_0} s(t)e^{-jn\omega_0 t}\,dt$$

$$= \frac{1}{3}\left[\int_0^2 -Ae^{-jn\omega_0 t}dt + \int_2^3 A\sin\pi(t-2)\,e^{-jn\omega_0 t}dt\right.$$

2-48(c). Cont'd Aside:

① $\int_0^2 -A e^{-jn\omega_0 t}\,dt = \left.\frac{-A e^{-jn2\pi t/3}}{-jn2\pi/3}\right|_0^2$

$= \frac{3A}{jn2\pi}\left(e^{-jn4\pi/3} - 1\right) = \frac{-j3A}{n2\pi}\left(e^{-jn4\pi/3} - 1\right)$

② $\int_2^3 A \sin[\pi(t-2)]\, e^{-jn2\pi t/3}\,dt =$

$\stackrel{*}{=} A \int_2^3 \sin(\pi t)\, e^{-jn2\pi t/3}\,dt$

From Sec. A-5 — Let $ax = -jn2\pi t/3 \Rightarrow a = -jn2/3$, $x = \pi t$

② $= \frac{A}{\pi}\frac{e^{a\pi t}}{1+a^2}\left[a\sin(\pi t) - \cos(\pi t)\right]\Big|_2^3$

$= \frac{A}{\pi(1+a^2)}\Big[e^{3a\pi}\big(a\overset{0}{\cancel{\sin 3\pi}} - \cos 3\pi\big) - e^{2a\pi}\big(a\overset{0}{\cancel{\sin 2\pi}} - \cos 2\pi\big)\Big]$

$= \frac{A}{(1+a^2)\pi}\left[e^{3a\pi} + e^{2a\pi}\right]$

$= \frac{A}{\pi(1-4n^2/9)}\left[\overset{1}{\cancel{e^{-jn2\pi}}} + e^{-jn4\pi/3}\right]$

$c_n = ① + ②$

$= \frac{A}{3\pi}\left[\frac{-j3}{2n}\left(e^{-jn4\pi/3} - 1\right) + \frac{\left(1 + e^{-jn4\pi/3}\right)}{(1 - 4n^2/9)}\right]$

$\Rightarrow \underline{\underline{c_n = \frac{A}{\pi}\left[\frac{-j}{2n}\left(e^{-jn4\pi/3} - 1\right) + \frac{\left(1 + e^{-jn4\pi/3}\right)}{(3 - 4n^2/3)}\right]}}$

$\underline{c_0 = V_{DC} = -0.454A}$

(d) $\underline{\underline{S(f) = \sum_{n=-\infty}^{\infty} c_n\,\delta(f - nf_0)}}$

Note: The DFT Computer Solution is given in P2-95.

**2-50** $s(t) = A_1 \cos(\omega_1 t + \phi_1) + A_2 \cos(\omega_2 t + \phi_2)$

$\phi_2 = 0$ for simplicity

(a.) $\omega_1 = \omega_2$ ; $\phi_1 = \phi_2 = 0$

$$s(t) = (A_1 + A_2) \cos \omega_1 t$$

$$s_{rms}(t) = \left[ (A_1 + A_2)^2 \frac{1}{T} \int_0^T \cos^2(\omega_1 t)\, dt \right]^{1/2}$$

$$= (A_1 + A_2) \left[ \frac{1}{2\pi} \int_0^{2\pi} \left( \frac{1}{2} + \frac{1}{2} \cos(2\theta) \right) d\theta \right]^{1/2}$$

$\omega t = \theta$

$dt = \frac{d\theta}{\omega_1} = \frac{d\theta\, T}{2\pi}$

$$= (A_1 + A_2) \left[ \frac{1}{2\pi} \left( \frac{1}{2} \right) 2\pi \right]^{1/2} = \frac{(A_1 + A_2)}{\sqrt{2}}$$

(b.) $\omega_1 = \omega_2$ ; $\phi_1 = \phi_2 + \pi/2 = \pi/2$

$$s(t) = A_1 \cos(\omega t + \pi/2) + A_2 \cos(\omega t)$$

$$= A_1 (0 - \sin \omega t \sin \pi/2) + A_2 \cos \omega t$$

$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$$= -A_1 \sin \omega t + A_2 \cos \omega t$$

$$\langle s^2(t) \rangle = \langle A_1^2 \sin^2(\omega t) \rangle - \langle A_1 A_2 \underbrace{\sin(\omega t) \cos(\omega t)}_{0 \text{ odd}} \rangle + \langle A_2^2 \cos^2(\omega t) \rangle = \frac{A_1^2 + A_2^2}{2}$$

$$\therefore\ s_{rms}(t) = \frac{\sqrt{(A_1^2 + A_2^2)}}{\sqrt{2}}$$

2-50. Cont'd

(c.) $\omega_1 = \omega_2$ ; $\phi_1 = \phi_2 + \pi = \pi$

$$s(t) = A_1 \cos(\omega t + \pi) + A_2 \cos \omega t$$

$$= (A_2 - A_1) \cos \omega t$$

$$\underline{\underline{s_{rms}(t) = \frac{(|A_2 - A_1|)}{\sqrt{2}}}} \quad \text{from (a.) above}$$

(d.) $\omega_1 = 2\omega_2$ ; $\phi_1 = \phi_2 = 0$

$$s(t) = A_1 \cos(2\omega_2 t) + A_2 \cos(\omega_2 t)$$

$$\langle s^2(t) \rangle = \langle A_1^2 \cos^2(2\omega_2 t) \rangle$$

$$+ A_1 A_2 \cancelto{0}{\langle \cos(2\omega_2 t) \cos(\omega_2 t) \rangle}$$

$$+ \langle A_2^2 \cos^2(\omega_2 t) \rangle$$

$$\therefore \underline{\underline{s_{rms}(t) = \frac{\sqrt{(A_1^2 + A_2^2)}}{\sqrt{2}}}}$$

(e.) $\omega_1 = 2\omega_2$ ; $\phi_1 = \phi_2 + \pi = \pi$

$$s(t) = A_1 \cos(2\omega_2 t + \pi) + A_2 \cos(\omega_2 t)$$

$$= -A_1 \cos(2\omega_2 t) + A_2 \cos(\omega_2 t)$$

$$\langle s^2(t) \rangle = \left( \frac{A_1^2 + A_2^2}{2} \right)$$

$$\underline{\underline{s_{rms}(t) = \frac{\sqrt{(A_1^2 + A_2^2)}}{\sqrt{2}}}}$$

**2-52** (a.) Over interval $(-4,4)$ $\perp$

$$\int_{-4}^{4} \phi_1^2(t)\,dt = 8$$

$$\int_{-4}^{4} \phi_2^2(t)\,dt = 8$$

$$\int_{-4}^{4} \phi_3^2(t)\,dt = 8$$

$$\int_{-4}^{4} \phi_1(t)\,\phi_2(t)\,dt = 4-4 = 0$$

$$\int_{-4}^{4} \phi_1(t)\,\phi_3(t)\,dt = -2+4-2 = 0$$

$$\int_{-4}^{4} \phi_2(t)\,\phi_3(t)\,dt = -2+2-2+2 = 0$$

(b.) $$\int_{-4}^{4} \phi_j(t)\,\phi_j^*(t)\,dt = 8 = K_j \quad j=1,3$$

$$\therefore\ \phi_j'(t) = \left\{\frac{\phi_j(t)}{\sqrt{8}}\right\} = \underline{\underline{\left\{\frac{\phi_j(t)}{2\sqrt{2}}\right\} \quad j=1,3}}$$

**2-53** $$w(t) = \frac{1}{2}\phi_1(t) - \frac{1}{2}\phi_2(t)$$

$$\underline{\underline{= \sqrt{2}\,\phi_1'(t) - \sqrt{2}\,\phi_2'(t)}}$$

**2-54** $$\epsilon = \int_{-4}^{4}\left[w(t) - \sum_{j=1}^{3} a_j\,\phi_j(t)\right]^2 dt$$

$$= \int_{-4}^{4}\left[w(t) - \frac{1}{2}\phi_1(t) + \frac{1}{2}\phi_2(t)\right] dt$$

$$\epsilon = \underline{\underline{0}}$$

**2-55** (e.) Cont'd $\quad a_j = \frac{1}{K_j}\int_a^b w(t)\,\phi_j^*(t)\,dt$

$$a_1 = \frac{1}{8}\int_{-4}^{4}\cos(\pi t/4)\,dt = \frac{1}{2\pi}\sin(\pi t/4)\Big|_{-4}^{4} = \underline{\underline{0}}$$

$$a_2 = \frac{1}{8}\left[\int_{-4}^{0}\cos(\pi t/4)\,dt - \int_{0}^{4}\cos(\pi t/4)\,dt\right]$$

$$= \frac{1}{2\pi}\left[\sin(\pi t/4)\Big|_{-4}^{0} - \sin(\pi t/4)\Big|_{0}^{4}\right]$$

$$= \frac{1}{2\pi}\{0-0\} = \underline{\underline{0}}$$

$$a_3 = \frac{1}{8}\left[\int_{-4}^{-2}-\cos\left(\frac{\pi t}{4}\right)dt + \int_{-2}^{2}\cos\left(\frac{\pi t}{4}\right)dt - \int_{2}^{4}\cos\left(\frac{\pi t}{4}\right)dt\right]$$

$$= \frac{1}{2\pi}\left[-\sin\left(\frac{\pi t}{4}\right)\Big|_{-4}^{-2} + \sin\left(\frac{\pi t}{4}\right)\Big|_{-2}^{2} - \sin\left(\frac{\pi t}{4}\right)\Big|_{2}^{4}\right]$$

$$= \frac{1}{2\pi}[1+2+1] = \underline{\underline{\frac{2}{\pi} = a_3}}$$

$$\therefore\ \underline{\underline{w(t) = \frac{2}{\pi}\phi_3(t)}} = \underline{\underline{\frac{4\sqrt{2}}{\pi}\phi_3'(t)}}$$

(Normalized (orthonormal))

**2-56**

$$\epsilon = \int_{-4}^{4}\left[w(t) - \sum_{j=1}^{3} a_j\,\phi_j(t)\right]^2 dt$$

2-56. Cont'd

$$\epsilon = \underbrace{\int_{-4}^{-2}\left[\cos\left(\frac{\pi t}{4}\right)+\frac{2}{\pi}\right]^2 dt}_{①} + \underbrace{\int_{-2}^{2}\left[\cos\left(\frac{\pi t}{4}\right)-\frac{2}{\pi}\right]^2 dt}_{②} + \underbrace{\int_{2}^{4}\left[\cos\left(\frac{\pi t}{4}\right)+\frac{2}{\pi}\right]^2 dt}_{③}$$

From Symmetry ② = ① and ③ = 2 · ①

$$\Rightarrow \epsilon = 4 \cdot ①$$

$$① = \int_{-4}^{-2}\left[\cos^2\left(\frac{\pi t}{4}\right)+\frac{4}{\pi}\cos\left(\frac{\pi t}{4}\right)+\frac{4}{\pi^2}\right]dt$$

$$= \frac{1}{2}\int_{-4}^{-2}\left[1+\cos\left(\frac{\pi t}{2}\right)\right]dt + \frac{16}{\pi^2}\sin\left(\frac{\pi t}{4}\right)\Big|_{-4}^{-2} + \frac{4}{\pi^2}t\Big|_{-4}^{-2}$$

$$= \frac{t}{2}\Big|_{-4}^{-2} + \frac{1}{\pi}\cancelto{0}{\sin\left(\frac{\pi t}{2}\right)}\Big|_{-4}^{-2} - \frac{16}{\pi^2} + \frac{8}{\pi^2} = 1-\frac{8}{\pi^2}$$

$$\Rightarrow ① = 0.189 \quad \Rightarrow \quad \epsilon = 4 \cdot ① = \underline{\underline{0.756 = \epsilon}}$$

$\{\phi_j(t)\}$ do not form a complete orthonormal set since they can represent only a subclass of possible waveforms.

**2-61**

$$c_n = \frac{1}{T}\int_{\tau_0}^{\tau_0+b} A\, e^{-jn\omega t}\, dt$$

$$= \frac{-A}{T}\,\frac{1}{jn\omega}\, e^{-jn\omega t}\Big|_{\tau_0}^{\tau_0+b}$$

$$= \frac{-A}{jn\omega T}\left(e^{-jn\omega(\tau_0+b)} - e^{-jn\omega\tau_0}\right)$$

$$= \frac{-A}{jn\omega T}\, e^{-jn\omega(\tau_0+\frac{b}{2})}\left(e^{-jn\frac{\omega b}{2}} - e^{jn\frac{\omega b}{2}}\right)$$

$$= \frac{2A}{n\omega T}\, e^{-jn\omega(\tau_0+\frac{b}{2})}\left(\frac{e^{jn\frac{\omega b}{2}} - e^{-jn\frac{\omega b}{2}}}{j2}\right)$$

$\omega = \frac{2\pi}{T}$

$$= \frac{A}{n\pi}\, e^{-jn\omega(\tau_0+\frac{b}{2})}\,\sin\left(\frac{n\pi b}{T}\right)$$

$$c_n = \frac{Ab}{T}\, e^{-jn\omega(\tau_0+\frac{b}{2})}\,\frac{\sin\left(\frac{n\pi b}{T}\right)}{n\pi b/T}$$

**2-64** Use (2-110) and (2-112)

(a.) $c_n = f_0\, P(f)\Big|_{f=nf_0}$

where $P(f) = \mathcal{F}[p(t)] = \int_{-\infty}^{\infty} p(t)\, e^{-j\omega t}\, dt$

For $f=0$

$$P(0) = \int_0^T A t\, dt = \frac{At^2}{2}\Big|_0^T = \frac{AT^2}{2}, \quad f=0$$

For $f \neq 0$

$$P(f) = \int_0^T A\, t\, e^{-j\omega t}\, dt$$

2-64. (a.) Cont'd

Let $u = At \quad dv = e^{-j\omega t}$
$du = A\,dt \quad v = e^{-j\omega t}/{-j\omega}$

$$P(f) = \left.\frac{Ate^{-j\omega t}}{-j\omega}\right|_0^T + \frac{A}{j\omega}\int_0^T e^{-j\omega t}\,dt$$

$$= \frac{jATe^{-j\omega T}}{\omega} + \frac{A}{\omega^2}\left(e^{-j\omega T} - 1\right)$$

$$P(f) = \frac{A\left[e^{-j\omega T} + j\omega T e^{-j\omega T} - 1\right]}{\omega^2}, \quad f \neq 0$$

$$c_n = \frac{1}{T_0} P\left(\omega = \frac{n2\pi}{T_0}\right) = f_0 P\left(\omega = n2\pi f_0\right)$$

$$c_n = \begin{cases} \dfrac{AT^2}{2T_0}, & n = 0 \\ \dfrac{A\left[e^{-j2\pi n f_0 T}(1 + jn2\pi f_0 T) - 1\right]}{T_0\omega^2}, & n \neq 0 \end{cases}$$

(b.) $x_n = Re\{c_n\} \quad ; \quad y_n = Im\{c_n\}$

$$c_n = \frac{A\left\{\left[\cos(n2\pi f_0 T) - j\sin(n2\pi f_0 T)\right]\cdot\left[1 + jn2\pi f_0 T\right] - 1\right\}}{T_0\omega^2}$$

2-64 (b.) Cont'd

$$x_n = \begin{cases} \dfrac{AT^2}{2T_0}, & n=0 \\ A\left\{\dfrac{\cos(n2\pi f_0 T) + n2\pi f_0 T \sin(n2\pi f_0 T) - 1}{T_0 \omega^2}\right\}, & n \neq 0 \end{cases}$$

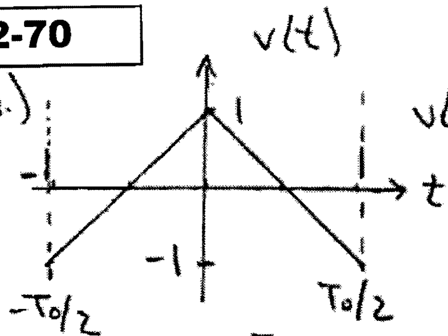
$$y_n = \begin{cases} 0, & n=0 \\ A\left\{\dfrac{n2\pi f_0 T \cos(n2\pi f_0 T) - \sin(n2\pi f_0 T)}{T_0 \omega^2}\right\}, & n \neq 0 \end{cases}$$

(c.)

$$D_n = \begin{cases} c_0, & n=0 \\ 2\sqrt{x_n^2 + y_n^2}, & n \geq 1 \end{cases}, \qquad \phi_n = \begin{cases} 0, & n=0 \\ \tan^{-1}\left(\dfrac{y_n}{x_n}\right), & n \geq 1 \end{cases}$$

**2-70**

(a.)




$$v(t) = \begin{cases} \dfrac{4}{T_0}\left(t + T_0/4\right), & -\frac{T_0}{2} < t < 0 \\ -\dfrac{4}{T_0}\left(t - T_0/4\right), & 0 < t < \frac{T_0}{2} \end{cases}$$

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} v(t)\, e^{-jn\omega_0 t}\, dt \qquad \underline{T_0 = 2}$$

$$= \frac{4}{T_0^2}\left[\int_{-T_0/2}^{0}\left(t + \frac{T_0}{4}\right)e^{-jn\omega_0 t}\, dt - \int_{0}^{T_0/2}\left(t - \frac{T_0}{4}\right)e^{-jn\omega_0 t}\, dt\right]$$

2-70(a) Cont'd

$$c_n = \frac{-8}{4n^2\pi^2}\left[\cos(n\omega_0 t) + n\omega_0 t \sin(n\omega_0 t)\right]\Big|_0^{T_0/2}$$

$$\ni c_n = \frac{2}{n^2\pi^2}\left\{1-(-1)^n\right\} = \begin{Bmatrix} 0, & n = \text{even} \\ \frac{4}{n^2\pi^2}, & n = \text{odd} \end{Bmatrix}$$

(b)
$$\langle v^2(t)\rangle = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} v^2(t)\,dt \underset{\text{For } v(t) = \text{even}}{=} \frac{2}{T_0}\int_0^{T_0/2} v^2(t)\,dt$$

$$= \frac{2}{T_0}\int_0^{T_0/2}\left[\frac{-4}{T_0}\left(t-\frac{T_0}{4}\right)\right]^2 dt = \frac{32}{T_0^3}\frac{\left(t-\frac{T_0}{4}\right)^3}{3}\Bigg|_0^{T_0/2}$$

$$= \frac{2\cdot 4^2}{3T_0^3}\left[\frac{2T_0^3}{4^3}\right] = \underline{\underline{\frac{1}{3}\text{ watt}}}$$

Computer Solution and comparison of results follows.

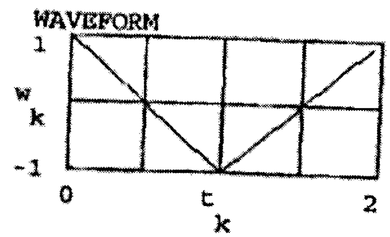
## 2-70. Cont'd

$M := 5 \qquad N := 2^M \qquad N = 32 \qquad k := 0 \,..\, N-1 \qquad T := 2$

$dt := \dfrac{T}{N} \qquad t_k := k \cdot dt$

$w_k := \mathrm{if}\left[t_k < 1, -2\left[t_k - 0.5\right], 2\left[t_k - 1.5\right]\right]$

$w_0 = 1 \qquad dt = 0.063$




(a.) Find the complex Fourier series.

$n := 0 \,..\, N-1 \qquad f_n := \dfrac{n}{T} \qquad fo := \dfrac{1}{T}$

From analytical computation,

$c_n := \mathrm{if}\left[\mathrm{mod}(n,2) \neq 0, \dfrac{4}{(n\,\pi)^2}, 0\right]$

Analytical values

FFT values

Alternately, computing FS using the FFT via (2-187),

$cc := \left[\dfrac{1}{\sqrt{N}}\right] \cdot \mathrm{icfft}(w)$

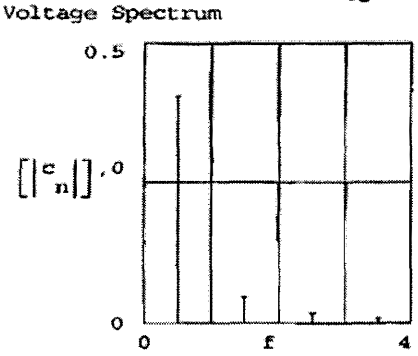
| $n$ | $c_n$ | $cc =$ |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0.405 | 0.407 |
| 2 | 0 | 0 |
| 3 | 0.045 | 0.046 |
| 4 | 0 | 0 |
| 5 | 0.016 | 0.018 |
| 6 | 0 | 0 |
| 7 | 0.008 | 0.01 |

(b)

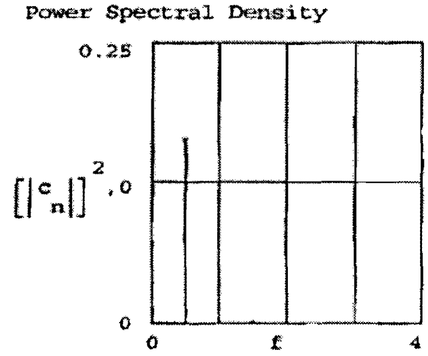
$P := 2 \sum_n c_n^2 - c_0^2 \qquad P = 0.333$

(c) and (d) $|V(f)| = \sum_{-\infty}^{\infty} |c_n|\, \delta(f - nf_0)\,, \quad \mathcal{P}(f) = \sum_{-\infty}^{\infty} |c_n|^2 \delta(f - nf_0)$

Voltage Spectrum




Power Spectral Density



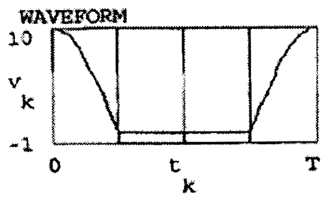
**2-72**

$M := 5 \qquad N := 2^M \qquad N = 32 \qquad k := 0..N-1 \qquad T := 1$

$dt := \frac{T}{N} \qquad t_k := k\,dt \qquad Sa(x) := \mathrm{if}\left[x \neq 0, \frac{\sin(x)}{x}, 1\right] \qquad \omega o := 2\frac{\pi}{T}$

$v_k := \mathrm{if}\left[\left|t_k - 0.5\,T\right| < 0.25\,T, 0, 10\cos\left[\omega o\, t_k\right]\right]$

$v_0 = 10 \qquad dt = 0.031$




Find the complex Fourier series.

$n := 0..N-1 \qquad f_n := \frac{n}{T} \qquad fo := \frac{1}{T}$

From analytical computation,

$c_n := 2.5\,(Sa(0.5\,(n+1)\,\pi) + Sa(0.5\,(n-1)\,\pi))$

Analytical results

Alternately, computing FS using the FFT via (2-187),

FFT results

$cc := \left[\frac{1}{\sqrt{N}}\right] \mathrm{icfft}(v)$

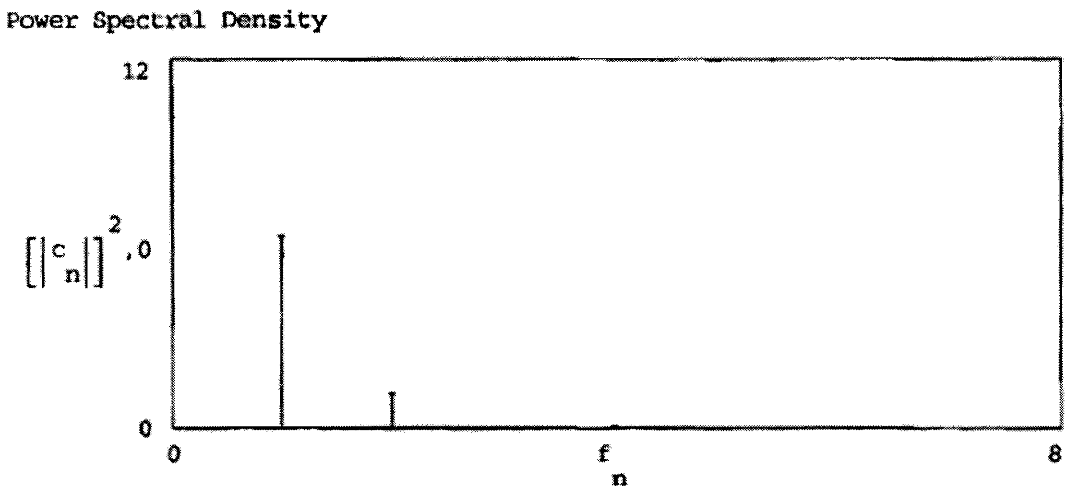
| $n$ | $c_n$ |
|---|---|
| 0 | 3.183 |
| 1 | 2.5 |
| 2 | 1.061 |
| 3 | 0 |
| 4 | -0.212 |
| 5 | 0 |
| 6 | 0.091 |
| 7 | 0 |
| 8 | -0.051 |
| 9 | 0 |
| 10 | 0.032 |
| 11 | 0 |
| 12 | -0.022 |
| 13 | 0 |
| 14 | 0.016 |
| 15 | 0 |
| 16 | -0.012 |
| 17 | 0 |
| 18 | 0.01 |
| 19 | 0 |
| 20 | -0.008 |
| 21 | 0 |
| 22 | 0.007 |
| 23 | 0 |
| 24 | -0.006 |
| 25 | 0 |

⋮

$$cc = \begin{bmatrix} 3.173 \\ 2.5 \\ 1.071 \\ 1.203\cdot 10^{-13} + 3.216\cdot 10^{-14}\,i \\ -0.223 \\ 6.336\cdot 10^{-14} + 3.226\cdot 10^{-14}\,i \\ 0.102 \\ -2.175\cdot 10^{-12} - 1.086\cdot 10^{-12}\,i \\ -0.062 \\ 2.705\cdot 10^{-14} + 3.207\cdot 10^{-14}\,i \\ 0.045 \\ 1.767\cdot 10^{-14} + 3.256\cdot 10^{-14}\,i \\ -0.036 \\ 9.774\cdot 10^{-15} + 3.232\cdot 10^{-14}\,i \\ 0.032 \\ -5.456\cdot 10^{-13} - 1.634\cdot 10^{-12}\,i \\ -0.031 \\ -3.22\cdot 10^{-15} + 3.235\cdot 10^{-14}\,i \end{bmatrix}$$

⋮

2-72. cont'd $\mathcal{P}(f) = \sum_{-\infty}^{\infty} |c_n|^2 \, \delta(f - nf_0)$




**2-80**

$$x(t) = e^{-400\pi t} \longleftrightarrow X(f) = \frac{1}{400\pi + j2\pi f}$$

$$\text{Energy in } x(t) = E_x = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_0^{\infty} e^{-800\pi t}\,dt = \frac{1}{800\pi} \text{ Joules}$$

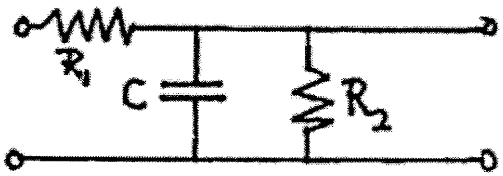
$$E_{out} = \frac{1}{2}E_x = \frac{1}{1600\pi} = 2\int_0^B |X(f)|^2 df = 2\int_0^B \frac{\frac{1}{4\pi^2}}{4\times10^4 + f^2}\,df = \frac{1}{2\pi}\left[\frac{1}{200}\tan^{-1}\left(\frac{B}{200}\right)\right]$$

$$\Rightarrow \frac{400\pi^2}{1600\pi} = \tan^{-1}\left(\frac{B}{200}\right) \Rightarrow \frac{B}{200} = \tan\left(\frac{\pi}{4}\right) = 1$$

$$\Rightarrow \underline{\underline{B = 200\text{ Hz}}}$$

**2-84**

$$C \| R_1 \Rightarrow Z_{\|} = \frac{R_2 \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{1 + j\omega R_2 C}$$




$$\Rightarrow H(f) = \frac{\frac{R_2}{1 + j\omega R_2 C}}{R_1 + \frac{R_2}{1 + j\omega R_2 C}} = \frac{R_2}{R_1 + R_2 + j\omega R_1 R_2 C}$$

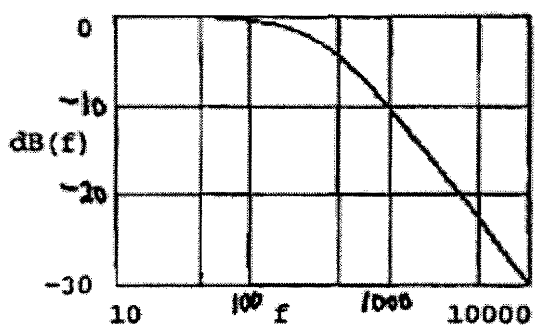
$R1 := 7.5 \cdot 10^{3}$ $\quad R2 := 15 \cdot 10^{3}$ $\quad C := 100\ 10^{-9}$

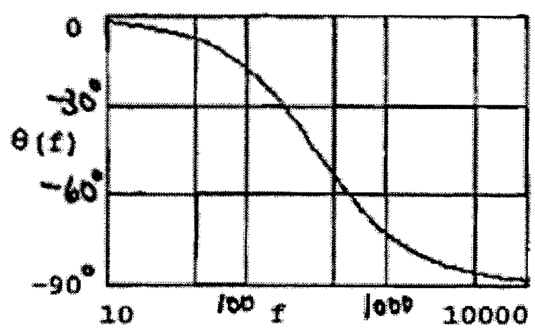
$f := 10, 20 \,..\, 10000$ $\quad j := \sqrt{-1}$ $\quad Hm := \frac{R2}{R1 + R2}$

$$H(f) := \frac{R2}{(R1 + R2) + j \cdot 2 \cdot \pi \cdot f \cdot R1 \cdot R2 \cdot C}$$

$Hm = 0.667$

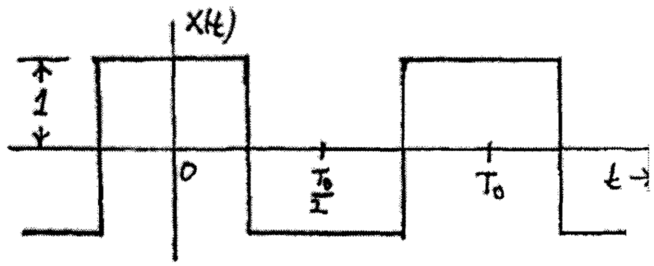
$$dB(f) := 20 \cdot \log\left[\frac{|H(f)|}{Hm}\right] \qquad \theta(f) := \left[\frac{180}{\pi}\right] \cdot \arg(H(f))$$




$$f3dB := \frac{R1 + R2}{2 \cdot \pi \cdot R1\ R2 \cdot C} \qquad f3dB = 318.31$$

**2-87**




Let the input square wave be represented by the Fourier series:

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

where
$$c_n = \begin{cases} \frac{2\sin(n\frac{\pi}{2})}{n\pi}, & n \neq 0 \\ 0, & n = 0 \end{cases}$$

for the waveform shown above.

Then the output waveform is, using (2-140),

$$y(t) = \sum_{-\infty}^{\infty} H(nf_0) c_n e^{jn\omega_0 t} = \sum_{-\infty}^{\infty} d_n e^{jn\omega_0 t}$$

where $d_n \triangleq H(nf_0)\, c_n$, $\quad H(f) = \frac{1}{1 + j(\frac{f}{f_1})}$

and $f_1 = 1{,}500$ Hz for the RC low-pass filter.

We also know that $d_{-n} = d_n^*$ since $x(t)$ is real and the impulse response of the filter is real.

Now reduce the output Fourier series to a form that can be easily plotted. Using (2-103),

$$y(t) = D_0 + \sum_{n=1}^{n=\infty} D_n \cos(n\omega_0 t + \phi_n)$$

where $D_0 = 0$ since $c_0 = 0$

and $D_n = 2|d_n| = 2|H(nf_0) c_n|$, $\quad n > 0$

2-87. Cont'd

or

$$\Rightarrow D_n = 2\left|\frac{1}{1+j\left(\frac{nf_0}{f_1}\right)}\right|\begin{Bmatrix}\frac{2}{n\pi}, & n=\text{odd}\\ 0, & n=\text{even}\end{Bmatrix} = \underline{\frac{4}{\sqrt{1+\left(\frac{nf_0}{f_1}\right)^2}\,(n\pi)},\ n=\text{odd}}$$

$$\phi_n = \underline{/d_n} = \underline{/2H(nf_0)c_n} = \underline{-\tan^{-1}\left(\frac{nf_0}{f_1}\right)+\pi\left(\frac{1-\sin\left(\frac{n\pi}{2}\right)}{2}\right),\ n=\text{odd}}$$

$$\Rightarrow y(t) = \underline{\underline{\sum_{\substack{n=1\\ n=\text{odd}}}^{\infty} D_n\cos(n\omega t+\phi_n)}}$$

The following MathCAD program plots this $y(t)$.

$f_o := 300$ $\quad f_1 := 1500$ $\quad n := 1,3\,..\,11$

$t := 0, 0.00005\,..\,0.004$

$$D_n := \frac{4}{n\,\pi\sqrt{1+\left[n\,\frac{f_o}{f_1}\right]^2}}$$

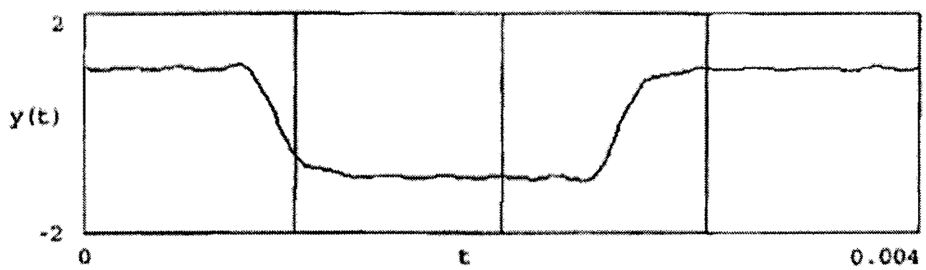
2-87. Cont'd.

$$\phi_n := \pi \left[ \frac{1 - \sin\left[n \frac{\pi}{2}\right]}{2} \right] - \operatorname{atan}\left[n \frac{fo}{f1}\right]$$

$$y(t) := \sum_n D_n \cos\left[n\, 2\, \pi\, fo\, t + \phi_n\right]$$

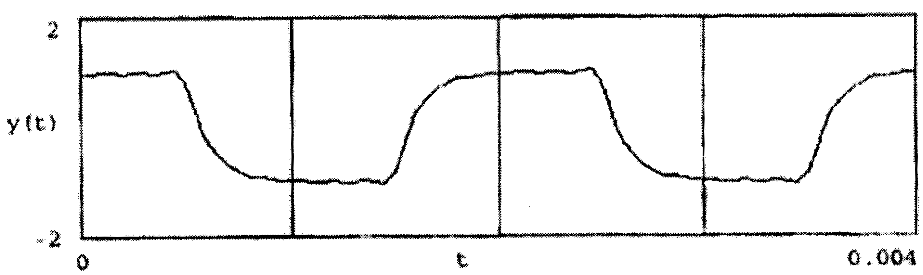
(a)

fo = 300



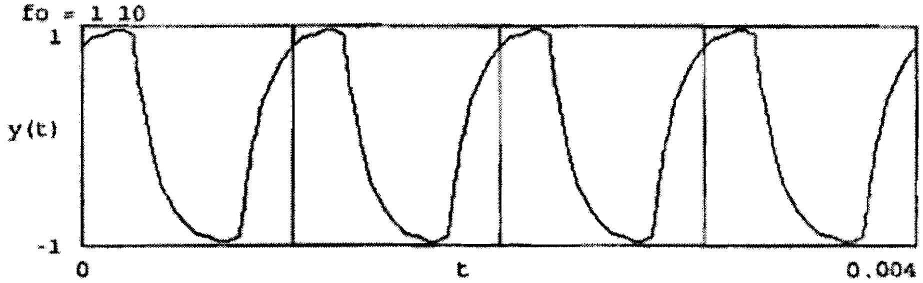

(b)

fo = 500




(c)

fo = $1 \cdot 10^3$



**2-90**

$$\omega_0 = 2\pi f_0 = 500 \Rightarrow f_0 = \frac{500}{2\pi}$$

$$f_s > 2f_0 = \frac{2(500)}{2\pi} = \frac{500}{\pi}$$

(a.) $$T_s = \frac{1}{f_s} \le \frac{\pi}{500} = \underline{\underline{6.28\ \text{msec}}}$$

(b.) $$N = \frac{1\ \text{sec}}{6.28\times 10^{-3}\ \text{sec/sample}} = \underline{\underline{160\ \text{samples}}}$$

**2-92**

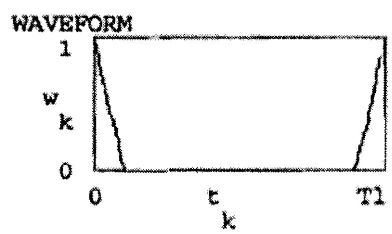
$M := 6 \qquad N := 2^M \qquad N = 64 \qquad k := 0 \,..\, N-1 \qquad T1 := 10 \qquad T := 1$

$dt := \frac{T1}{N} \qquad t_k := k\,dt$

NOTE: In FFT time domain, points for negative time are the same as those measured from the end of the data span-length T1 for positive time.

$$w_k := \text{if}\left[t_k < T, \left[\frac{-1}{T}\right]\left[t_k - T\right], 0\right] + \text{if}\left[t_k > (T1 - T), \frac{t_k - (T1 - T)}{T}, 0\right]$$

$w_0 = 1 \qquad dt = 0.156$

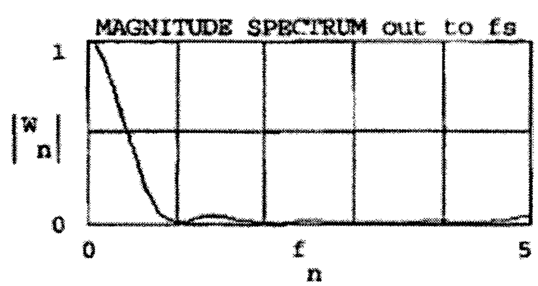



$n := 0 \,..\, N-1$

$W := dt\left[\sqrt{N}\right]\ \text{icfft}(w)$

$f_n := \frac{n}{T1} \qquad fs := \frac{1}{dt}$

$f_1 = 0.1 \qquad fs = 6.4$



**2-100** $s(t) = \Lambda\left(\frac{t}{T_0}\right) \leftrightarrow S(f) = T_0\left[S_a(\pi f T_0)\right]^2$

Table 2-2

(a.) Using results in 2-61 (1.) above $\Rightarrow$ $\underline{\underline{B_{abs} = \infty}}$

(b.) $S(f_{3dB}) = \frac{T_0}{\sqrt{2}} = T_0\left[S_a(\pi f_{3dB} T_0)\right]^2$

$\Rightarrow \pi f_{3dB} T_0 \approx (2)^{1/4} \Rightarrow$ $\underline{\underline{B_{3dB} = f_{3dB} = \frac{1.19}{\pi T_0} = 0.38/T_0}}$

(c.) $B_{eq} = \frac{1}{|H(f_0)|^2}\int_0^\infty |H(f)|^2 df = \frac{1}{T_0^2}\int_0^\infty T_0^2\left[S_a(\pi f T_0)\right]^4 df$

$= \frac{1}{\pi T_0}\left(\frac{\pi}{3}\right) = \frac{1}{3T_0}$ $\Rightarrow$ $\underline{\underline{B_{eq} = \frac{1}{3T_0}}}$

(d.) $B_{zero\ cross} = \frac{1}{T_0}$ (Similar to 2-90 (4) above)