

Chapter 5

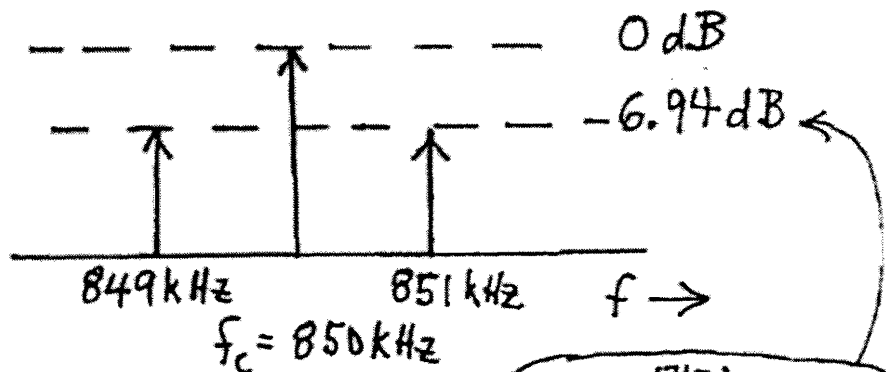
5-1

$$(a.) \text{ dBk} = 10 \log_{10} \left(\frac{5000}{1000} \right) = \underline{\underline{6.99 \text{ dBk}}}$$

$$(b.) P = \frac{A_c^2}{2R} \Rightarrow A_c = \sqrt{2PR} = \sqrt{2(5000)(50)} = \underline{\underline{707 \text{ volts}}}$$

$$\underline{\underline{s(t) = 707 [1 + 0.9 \cos(2000\pi t)] \cos(2\pi 850000t)}}$$

$$(c.) s(t) = 707 \cos \omega_c t + \frac{0.9(707)}{2} \cos[(\omega_c - \omega_m)t] + \frac{0.9(707)}{2} \cos[(\omega_c + \omega_m)t]$$



$$20 \log_{10} \left(\frac{318}{707} \right) = -6.94$$

$$(d.) P_{\text{AVE}} = \frac{(707)^2}{2(50)} + \frac{(318)^2}{2(50)} + \frac{(318)^2}{2(50)} = \underline{\underline{7.021 \text{ kW}}}$$

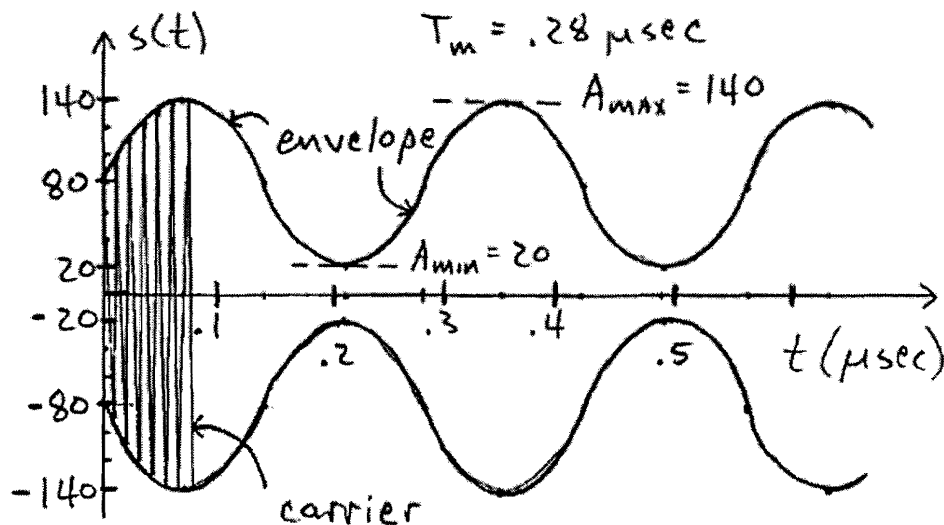
$$(e.) P_{\text{EP}} = \frac{[(707)(1.9)]^2}{2(50)} = \underline{\underline{18.045 \text{ kW}}}$$

5-4

(a.) $m(t) = -0.2 + 0.6 \sin \omega_m t$

$f_m = f_i = 3.57 \text{ MHz} ; A_c = 100$

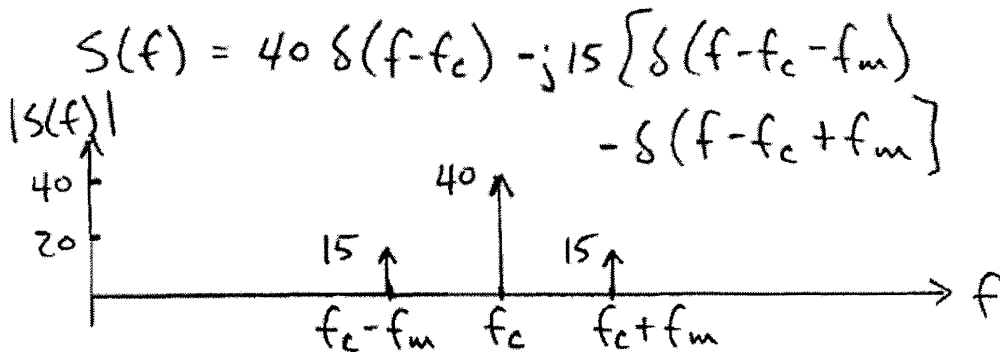
$s(t) = 100 (0.8 + 0.6 \sin \omega_m t) \cos \omega_c t$



(b.) $\% \text{ pos. mod.} = \frac{A_{\max} - A_c}{A_c} (100) = \frac{140 - 100}{100} (100)$
 $= 40\%$

$\% \text{ neg. mod.} = \frac{A_c - A_{\min}}{A_c} (100) = \frac{100 - 20}{100} (100)$
 $= 80\%$

(c.) $f > 0$



5-6

From (5-5a)

$$\% \text{ Pos. Mod.} = \frac{A_{\max} - A_c}{A_c} (100) \stackrel{\text{given}}{=} 120$$

where:

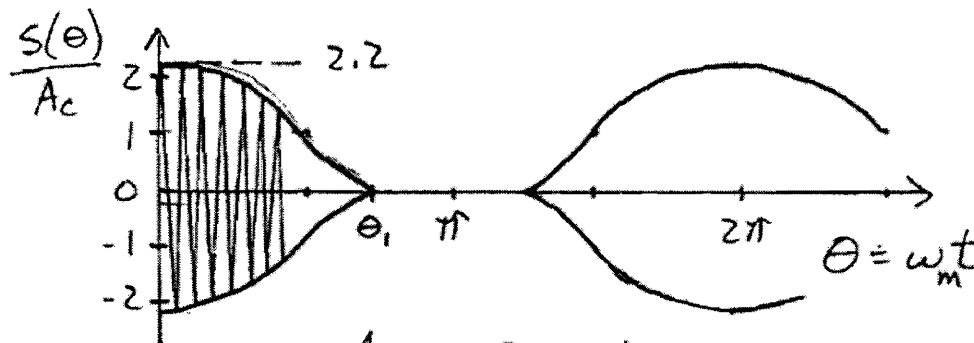
$$s(t) = \begin{cases} A_c \{1 + A_m \cos \omega_m t\} \cos \omega_c t; & m(t) \geq -1 \\ 0 & ; m(t) < -1 \end{cases}$$

$$m(t) = A_m \cos \omega_m t \quad ; \quad A_{\max} = A_c [1 + A_m]$$

$$\frac{A_{\max} - A_c}{A_c} = \underline{\underline{A_m = 1.2}}$$

$$g(t) = \begin{cases} A_c \{1 + 1.2 \cos \omega_m t\}, & 1.2 \cos \omega_m t \geq -1 \\ 0 & , 1.2 \cos \omega_m t < -1 \end{cases}$$

$$= \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_m t} \Rightarrow G(f) = \sum_{n=-\infty}^{\infty} c_n \delta(f - n f_m)$$



$$A_m \cos \theta_1 = -1$$

Aside: $\theta_1 = \cos^{-1}\left(\frac{-1}{1.2}\right) = \underline{146.4^\circ}$

$$c_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g(t) e^{-jn\omega_m t} dt$$

$$= \frac{A_c}{2\pi} \int_{-\theta_1}^{\theta_1} [1 + A_m \cos \theta] e^{-jn\theta} d\theta$$

5-6. Cont'd

$$c_n = \frac{A_c}{2\pi} \left[\frac{e^{-jn\theta}}{-jn} \right]_{-\theta_1}^{\theta_1} + A_m \int_{-\theta_1}^{\theta_1} (\cos \theta) e^{-jn\theta} d\theta \right]$$

$$= \frac{A_c}{2\pi} \left[\frac{z}{n} \left(\frac{e^{jn\theta_1} - e^{-jn\theta_1}}{jz} \right) + A_m \frac{e^{-jn\theta}}{(-jn)^2 + 1} \right]$$

Using Sec. A-5
where $a = -jn$

$$\cdot \left(-jn \cos \theta + \sin \theta \right) \Big|_{-\theta_1}^{\theta_1} \right]$$

$$= \frac{A_c}{2\pi} \left[\frac{2 \sin n\theta_1}{n} + A_m \left\{ \frac{e^{-jn\theta_1} (-jn \cos \theta_1 + \sin \theta_1)}{1 - n^2} \right. \right.$$

$$\left. - \frac{e^{jn\theta_1} (-jn \cos \theta_1 - \sin \theta_1)}{1 - n^2} \right\} \right]$$

$$= \frac{A_c}{2\pi} \left[2\theta_1 \left(\frac{\sin n\theta_1}{n\theta_1} \right) + A_m \left\{ \frac{jn(zj) \left(\frac{e^{jn\theta_1} - e^{-jn\theta_1}}{zj} \right) \cos \theta_1}{1 - n^2} \right. \right.$$

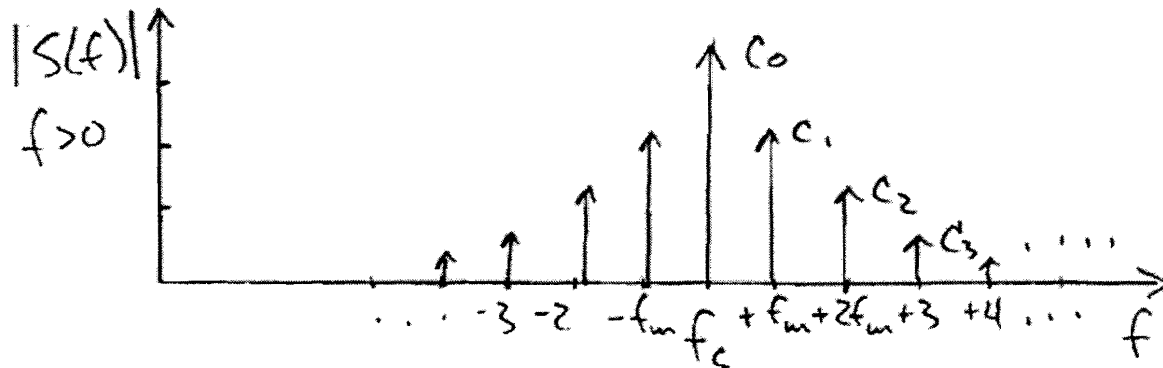
$$\left. + \frac{z \left(\frac{e^{jn\theta_1} + e^{-jn\theta_1}}{z} \right) \sin \theta_1}{1 - n^2} \right\} \right]$$

$A_m = 1.2$
 $\theta_1 = 146.4^\circ$

$$c_n = \frac{A_c}{2\pi} \left[2\theta_1 \left(\frac{\sin(n\theta_1)}{n\theta_1} \right) + 2A_m \left\{ \frac{\cos(n\theta_1) \sin \theta_1}{1 - n^2} - \frac{n \sin(n\theta_1) \cos \theta_1}{1 - n^2} \right\} \right]$$

$$S(f) = \frac{1}{2} \left[\sum_{-\infty}^{\infty} c_n \delta(f - f_c - n f_m) + \sum_{-\infty}^{\infty} c_n^* \delta(-f - f_c - n f_m) \right]$$

5-6 . Cont'd $c_n = c_n^*$ (c_n real)

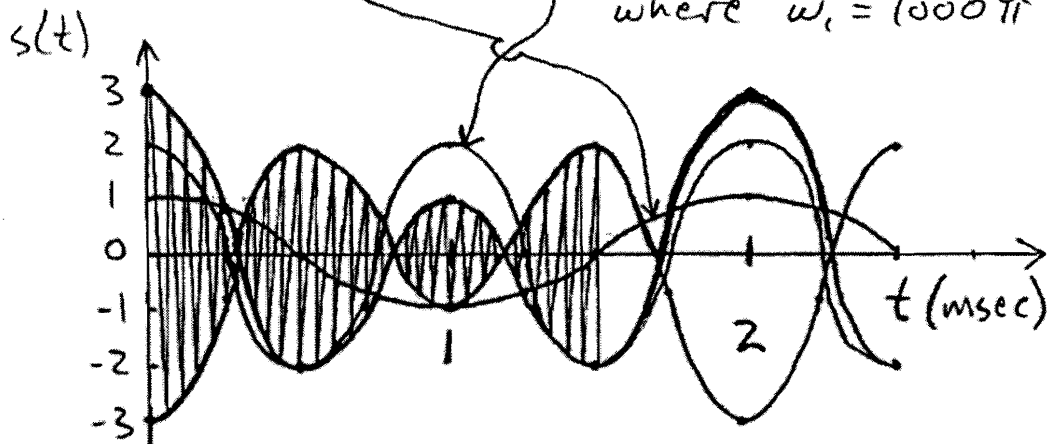


$$\begin{aligned} c_0 &= \frac{A_c}{2\pi} \left[2\theta_1 (\text{rad.}) + 2A_m \sin \theta_1 \right] \\ &= \frac{A_c}{2\pi} \left[2(7.56) + 2(1.2)(.553) \right] \end{aligned}$$

5-10 (a.) DSB-SC $m(t) = \cos \omega_1 t + 2 \cos 2\omega_1 t$

$$s(t) = \left[\cos \omega_1 t + 2 \cos 2\omega_1 t \right] \cos \omega_c t$$

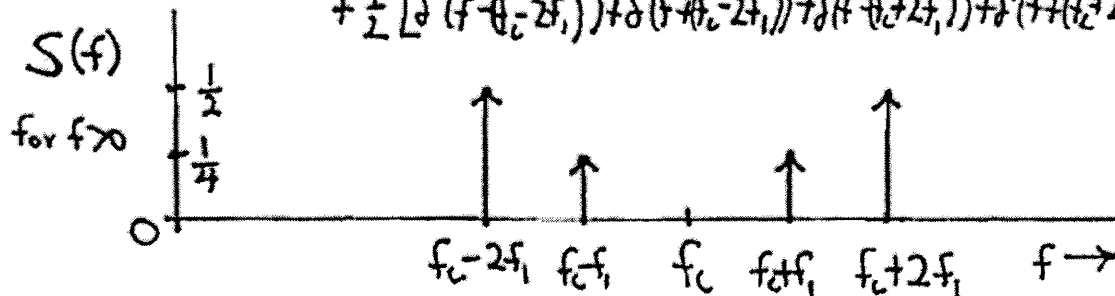
where $\omega_1 = 1000\pi$



$$\begin{aligned} \text{(b.) } s(t) &= \frac{1}{2} \left[\cos(\omega_c - \omega_1)t + \cos(\omega_c + \omega_1)t \right] \\ &\quad + \cos(\omega_c - 2\omega_1)t + \cos(\omega_c + 2\omega_1)t \end{aligned}$$

5-10 (b) Cont'd $S(-f) = S(f)$ even

$$S(f) = \mathcal{F}[s(t)] = \frac{1}{4} [\delta(f - (f_c - f_1)) + \delta(f + (f_c - f_1)) + \delta(f - (f_c + f_1)) + \delta(f + (f_c + f_1))] \\ + \frac{1}{2} [\delta(f - (f_c - 2f_1)) + \delta(f + (f_c - 2f_1)) + \delta(f - (f_c + 2f_1)) + \delta(f + (f_c + 2f_1))]$$



$$(c.) P_{AV} = \frac{1}{2} \left[\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + (1)^2 + (1)^2 \right] = \underline{\underline{1.25 W}}$$

$$(d.) A_{max} = 3 \Rightarrow PEP = \frac{(3)^2}{2} = \underline{\underline{4.5 W}}$$

5-15

$$v_1(t) = \cos(2\pi(\frac{1}{2}B)t) = \cos(\pi Bt)$$

$$v_3(t) = m(t)v_1(t) = m(t)\cos(\pi Bt) \leftrightarrow V_3(f) = \frac{1}{2} [M(f - \frac{1}{2}B) + M(f + \frac{1}{2}B)]$$

$$v_4(t) = m(t)v_2(t) = m(t)\sin(\pi Bt) \leftrightarrow V_4(f) = \frac{1}{2}j [-M(f - \frac{1}{2}B) + M(f + \frac{1}{2}B)]$$

$$V_5(f) = \begin{cases} V_3(f), & |f| < \frac{1}{2}B \\ 0, & \text{elsewhere} \end{cases} = \begin{cases} \frac{1}{2} [M(f - \frac{1}{2}B) + M(f + \frac{1}{2}B)], & |f| < \frac{1}{2}B \\ 0, & \text{if elsewhere} \end{cases}$$

$$\text{Likewise } V_6(f) = \begin{cases} \frac{1}{2}j [-M(f - \frac{1}{2}B) + M(f + \frac{1}{2}B)], & |f| < \frac{1}{2}B \\ 0, & \text{if elsewhere} \end{cases}$$

$$v_9(t) = v_5(t) \cos[2\pi(f_c + \frac{1}{2}B)t]$$

$$\Rightarrow V_9(f) = \frac{1}{2} [V_5(f - f_c - \frac{1}{2}B) + V_5(f + f_c + \frac{1}{2}B)] \\ = \frac{1}{4} \begin{cases} [M(f - f_c - \frac{1}{2}B - \frac{1}{2}B) + M(f - f_c - \frac{1}{2}B + \frac{1}{2}B)], & |f - f_c - \frac{1}{2}B| < \frac{1}{2}B \\ [M(f + f_c + \frac{1}{2}B - \frac{1}{2}B) + M(f + f_c + \frac{1}{2}B + \frac{1}{2}B)], & |f + f_c + \frac{1}{2}B| < \frac{1}{2}B \\ 0, & \text{if elsewhere} \end{cases}$$

Aside: $|f - f_c - \frac{1}{2}B| < \frac{1}{2}B \Rightarrow -\frac{1}{2}B < f - f_c - \frac{1}{2}B < \frac{1}{2}B$
 $\Rightarrow f_c + \frac{1}{2}B - \frac{1}{2}B < f < f_c + \frac{1}{2}B + \frac{1}{2}B \Rightarrow f_c < f < f_c + B$
 Likewise $|f_c + f_c + \frac{1}{2}B| < \frac{1}{2}B \Rightarrow -f_c - B < f < -f_c$

Thus,

$$V_9(f) = \begin{cases} \frac{1}{4} [M(f - f_c - B) + M(f - f_c)], & f_c < f < f_c + B \\ \frac{1}{4} [M(f + f_c) + M(f + f_c + B)], & -f_c - B < f < -f_c \\ 0, & f \text{ elsewhere} \end{cases}$$

Likewise

$$V_{10}(f) = \frac{1}{2}j [-V_6(f - f_c - \frac{1}{2}B) + V_6(f + f_c + \frac{1}{2}B)]$$

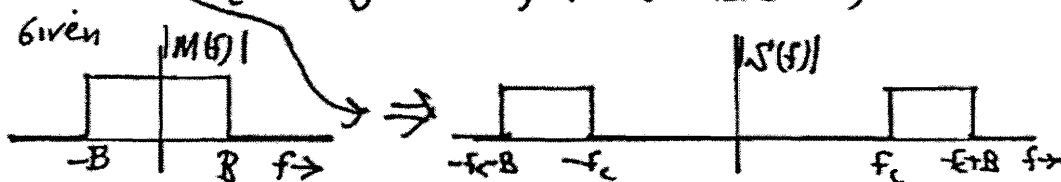
$$= \begin{cases} \frac{1}{4} [-M(f - f_c - \frac{1}{2}B - \frac{1}{2}B) + M(f - f_c - \frac{1}{2}B + \frac{1}{2}B)], & |f - f_c - \frac{1}{2}B| < \frac{1}{2}B \\ \frac{1}{4} [M(f + f_c + \frac{1}{2}B - \frac{1}{2}B) - M(f + f_c + \frac{1}{2}B + \frac{1}{2}B)], & |f + f_c + \frac{1}{2}B| < \frac{1}{2}B \\ 0, & f \text{ elsewhere} \end{cases}$$

$$\Rightarrow V_{10}(f) = \begin{cases} \frac{1}{4} [-M(f - f_c - B) + M(f - f_c)], & f_c < f < f_c + B \\ \frac{1}{4} [M(f + f_c) - M(f + f_c + B)], & -f_c - B < f < -f_c \\ 0, & f \text{ elsewhere} \end{cases}$$

$$\text{Output} = s(t) = u_9(t) + u_{10}(t)$$

$$\Rightarrow S(f) = V_9(f) + V_{10}(f)$$

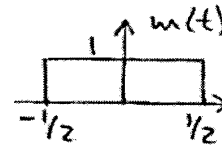
$$\Rightarrow S(f) = \begin{cases} \frac{1}{2} M(f - f_c), & f_c < f < f_c + B \\ \frac{1}{2} M(f + f_c), & -f_c - B < f < -f_c \\ 0, & f \text{ elsewhere} \end{cases} = \text{USSB}$$



$\Rightarrow s(t)$ is a USSB signal QEB

5-17

$$m(t) = \begin{cases} 1, & |t| < 1/2 \\ 0, & \text{elsewhere} \end{cases}$$



$$(a.) \quad \hat{m}(t) = m(t) * \frac{1}{\pi t}$$

$$= \int_{-1/2}^{1/2} \frac{1}{\pi} \frac{1}{t-\lambda} d\lambda = \frac{-1}{\pi} \int_{t+1/2}^{t-1/2} \frac{1}{\lambda_1} d\lambda_1$$

$$\lambda_1 = t - \lambda$$

$$d\lambda_1 = -d\lambda$$

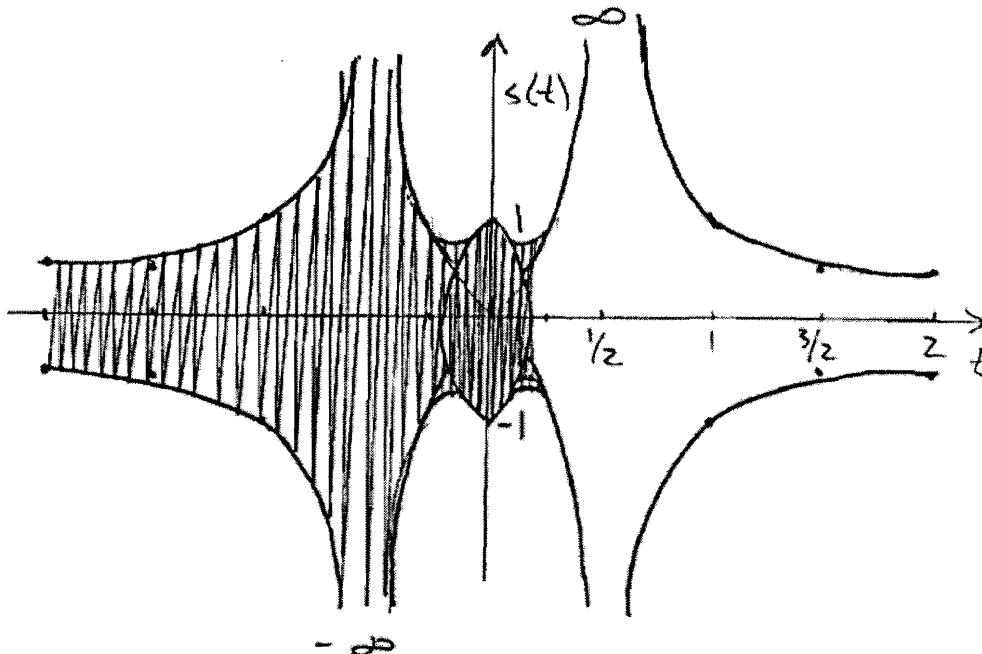
$$= \int_{t-1/2}^{t+1/2} \frac{1}{\pi} \frac{1}{\lambda_1} d\lambda_1 = \frac{1}{\pi} \left(\ln |\lambda_1| \right) \Big|_{t-1/2}^{t+1/2}$$

$$\hat{m}(t) = \frac{1}{\pi} \ln \left[\frac{|t+1/2|}{|t-1/2|} \right]$$

(b.) For USSB:

$$s(t) = m(t) \cos \omega_c t - \hat{m}(t) \sin \omega_c t$$

$$= m(t) \cos \omega_c t - \frac{1}{\pi} \ln \left[\frac{|t+1/2|}{|t-1/2|} \right] \sin \omega_c t$$



$$(c.) \quad \underline{s(t)_{\max} = \infty}$$

5-19 Note: T has units of Hz.

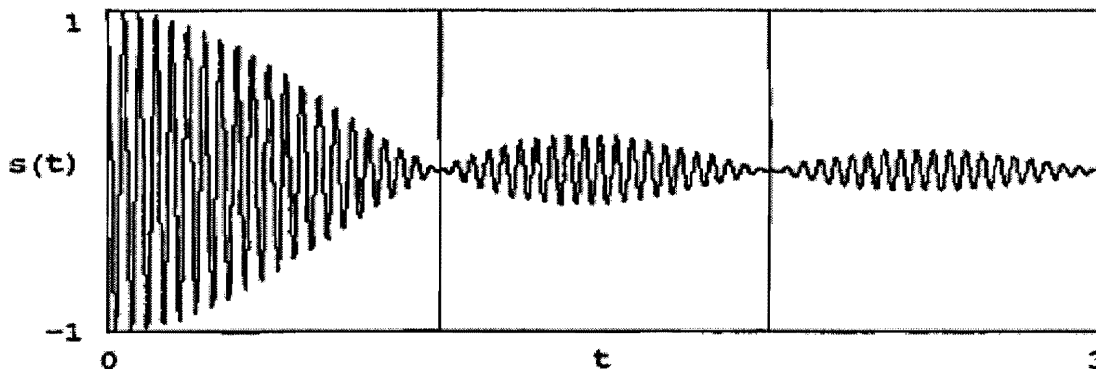
$$(a.) \quad m(t) = \frac{\sin(\pi T t)}{\pi T t} \longleftrightarrow M(f) = \frac{1}{T} \Pi\left(\frac{f}{T}\right) = \frac{1}{T} \left[\Pi\left(\frac{f - \frac{T}{4}}{T/2}\right) + \Pi\left(\frac{f + \frac{T}{4}}{T/2}\right) \right]$$

$$\mathcal{F}[h(t)] = M(f) \begin{cases} -j, & f > 0 \\ j, & f < 0 \end{cases} = \frac{1}{T} \left[-j \Pi\left(\frac{f - T/4}{T/2}\right) + j \Pi\left(\frac{f + T/4}{T/2}\right) \right]$$

$$\begin{aligned} h(t) &= -j \frac{1}{2} \frac{\sin(\pi T t)}{\pi T t} e^{j 2 \pi \frac{T}{4} t} + j \frac{1}{2} \frac{\sin(\pi T t)}{\pi T t} e^{-j 2 \pi \frac{T}{4} t} \\ &= \frac{\sin(\pi T t)}{\pi T t} \frac{e^{j \pi T t} - e^{-j \pi T t}}{2j} = \frac{\sin(\pi T t)}{\pi T t} \sin(\pi T t) = \frac{\sin^2(\pi T t)}{\pi T t} \end{aligned}$$

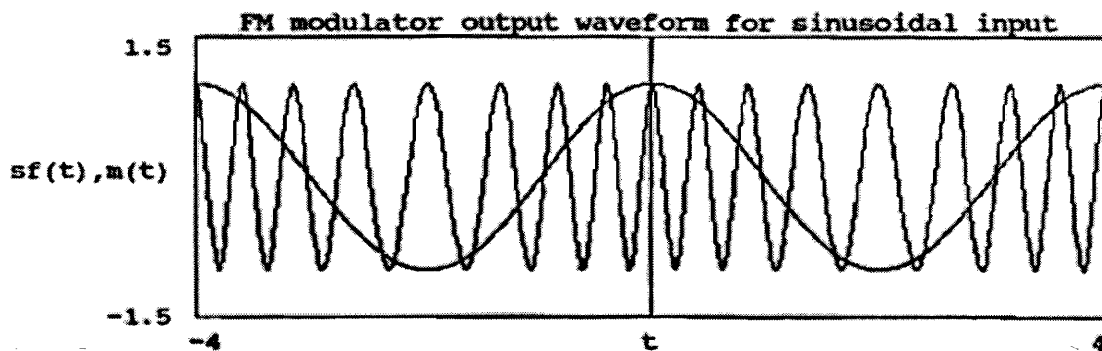
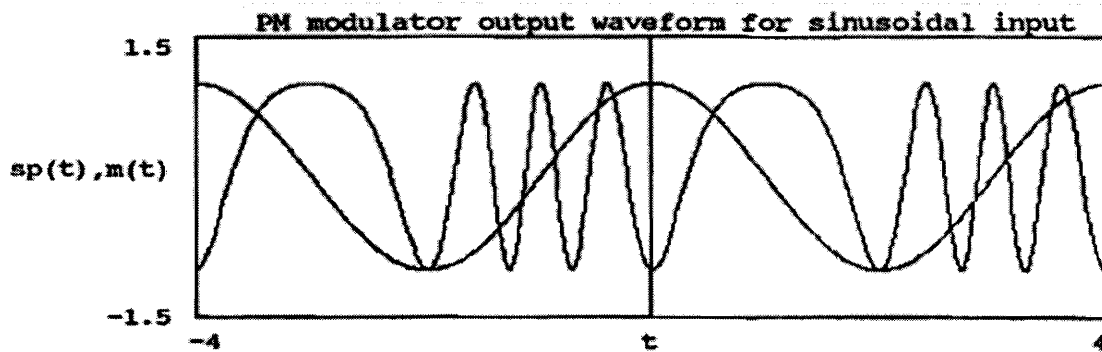
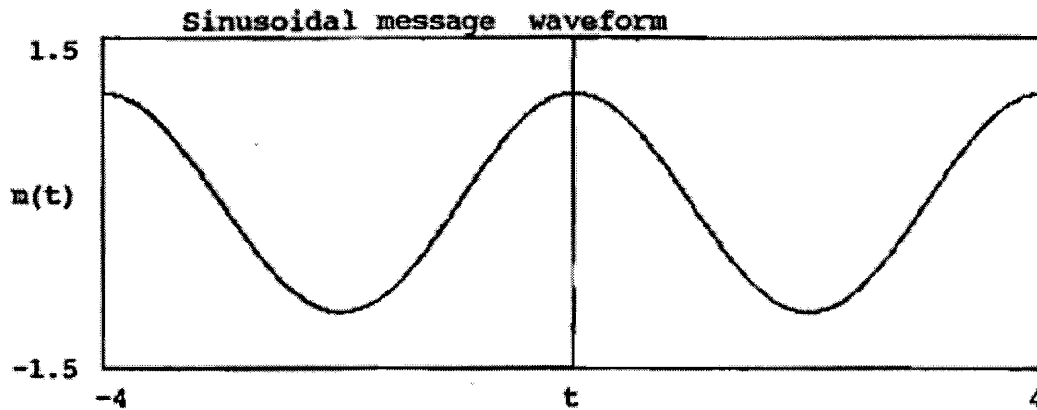
$$\begin{aligned} (b) \quad & t := 10^{-7}, 0.002 \dots 3 \quad T := 2 \\ & f_c := 20 \quad \omega_c := 2 \cdot \pi \cdot f_c \\ & m(t) := \frac{\sin(\pi \cdot T \cdot t)}{\pi \cdot T \cdot t} \quad m_h(t) := \frac{\left[\sin\left[\pi \cdot \frac{T}{2} \cdot t\right] \right]^2}{\pi \cdot \frac{T}{2} \cdot t} \end{aligned}$$

$$s(t) := m(t) \cdot \cos\left[\omega_c \cdot t\right] - m_h(t) \cdot \sin\left[\omega_c \cdot t\right]$$



5-27

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t := -5, -4.99 .. 5    fc := 1  fm := 0.25 fc    Dp := π    Df := π
m(t) := cos(2·π fm·t)
θ(t) := ⌊  $\frac{Df}{2 \cdot \pi \cdot fm}$  ⌋ · sin(2 π fm·t)
sp(t) := cos(2 π·fc·t + Dp·m(t))
sf(t) := cos(2·π·2·fc·t + θ(t))
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5-30

$$(a.) f_{BPF} = \frac{103.7}{8} \text{ MHz} = \underline{\underline{12.96 \text{ MHz}}}$$

$$\Delta F_{BPF} = \frac{75 \text{ KHz}}{8} = 9.375 \text{ KHz}$$

$$BW_{BPF} = 2(\Delta F + f_m) = 2(9.375 + 15) \text{ KHz} \\ = \underline{\underline{48.75 \text{ KHz}}}$$

$$(b.) f_{BPF} = f_c + f_o \Rightarrow f_o = 12.96 - 5 = \underline{\underline{7.96 \text{ MHz}}} \\ f_{BPF} = f_c - f_o \Rightarrow f_o = 12.96 + 5 = \underline{\underline{17.96 \text{ MHz}}} \\ \quad \quad \quad \uparrow \\ \quad \quad \quad f_c = 5 \text{ MHz}$$

$$(c.) \Delta F_{FME} = \frac{75 \text{ KHz}}{8} = \underline{\underline{9.38 \text{ KHz}}}$$

5-32 (a.) $\Theta(t) = \mathcal{D}_p m_p(t) = 20 \cos \omega_1 t$

$\Rightarrow m_p(t) = \frac{20}{D_p} \cos \omega_1 t = \underline{0.2 \cos(2000\pi t)}$, $m_p(t)_{peak} = \underline{0.2V}$, $f_m = \underline{1 \text{ kHz}}$

(b) $\Theta(t) = \mathcal{D}_f \int_{-\infty}^t m_f(\lambda) d\lambda = 20 \cos \omega_1 t$

$\Rightarrow m_f(t) = \frac{20}{D_p} \frac{d}{dt} [\cos \omega_1 t] = \frac{-20}{10^6} (2000\pi) \sin \omega_1 t$

$m_f(t) = \underline{-0.1257 \sin \omega_1 t}$, $m_f(t)_{peak} = \underline{0.1257V}$, $f_m = \underline{1 \text{ kHz}}$

(c) $P_{Av} = \frac{V_{rms}^2}{R} = \frac{(500)^2}{2(50)} = \underline{2.5 \text{ kW}}$

$PEP = \underline{2.5 \text{ kW}}$

5-37 (a.) $P_T = \frac{1}{2} \langle |g(t)|^2 \rangle = \frac{A_c^2}{2} = \frac{(100)^2}{2} = \underline{5,000 \text{ watts}}$

(b.) $P_T = \frac{1}{2} \langle |g(t)|^2 \rangle = \frac{1}{2} \langle \sum_{n=-N}^N |c_n|^2 \rangle = \frac{1}{2} \sum_{n=-N}^N |c_n|^2 = \frac{1}{2} A_c^2 \sum_{n=-N}^N J_n^2(\beta)$
Using (5-55) Using (5-57)

$\Rightarrow P_T = \frac{1}{2} A_c^2 \left[J_0^2(\beta) + 2 \sum_{n=1}^N J_n^2(\beta) \right]$

where $2Nf_m = B_T \Rightarrow N = \frac{B_T}{2f_m} = \frac{56 \text{ Hz}}{2(8)} = 3.5 \Rightarrow \text{Use } N=3$

Also, $\beta = \frac{\Delta F}{f_m} = \frac{k_d A_m}{f_m} = \frac{(8 \text{ Hz/volt})(5 \text{ volt})}{8 \text{ Hz}} = 5.0$

Using MathCAD: $Ac := 100$ $\beta := 5.0$ $N := 3$ $n := 1 \dots N$

$PT := 0.5 \cdot Ac^2 \left[J_0(\beta)^2 + 2 \cdot \sum_n J_n(n, \beta)^2 \right]$

$PT = 2583.485 \text{ watts (normalized for } 1\Omega)$

5-38

$$s(t) = A_c \cos[\omega_c t + \beta \sin \omega_m t]$$

where $A_c = 1$, $\omega_c = 2\pi(146.52 \times 10^6)$

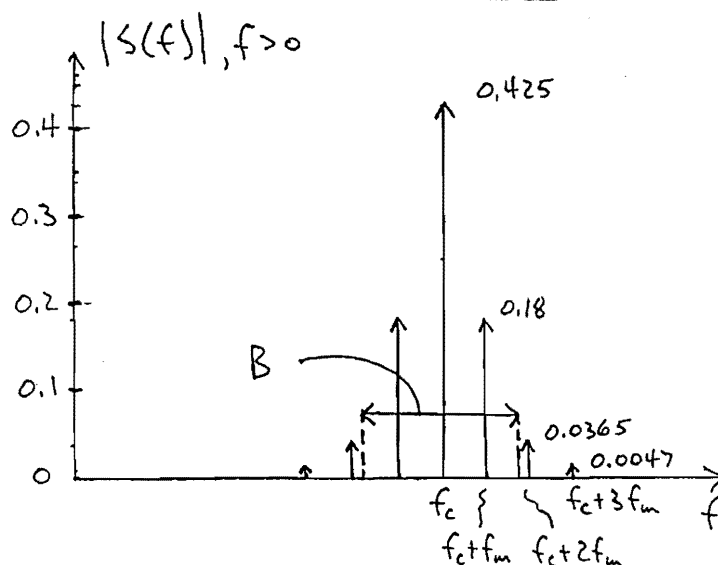
$$\omega_m = 2\pi(10^3), \quad \beta = 45^\circ \left(\frac{\pi \text{ radians}}{180^\circ} \right) = 0.7854 \text{ rad.}$$

$$(5-60) \quad G(f) = \sum_{n=-\infty}^{\infty} J_n(\beta) \delta(f - n f_m) \quad (\delta(-f) = \delta(f))$$

$$S(f) = \frac{1}{2} [G(f - f_c) + G^*(-f - f_c)]$$

$$= \frac{1}{2} \left\{ \sum_{n=-\infty}^{\infty} J_n(\beta) [\delta(f - n f_m - f_c) + \delta(f - n f_m + f_c)] \right\}$$

n	$J_n(\beta)$	Carson's Rule BW:
0	0.85	
1	0.36	$B = 2 f_m (\beta + 1)$
2	0.073	$= 2(1 \text{ KHz})(1.7854)$
3	0.0094	$= \underline{\underline{3.57 \text{ KHz}}}$



$$\text{Total } P_{Av} = \frac{A_c^2}{2} = 0.5$$

$$P_{av}(\text{within } B) = \frac{(0.85)^2 + 2(0.36)^2}{2} = \frac{0.9817}{2} = \underline{\underline{98.17\%}}$$

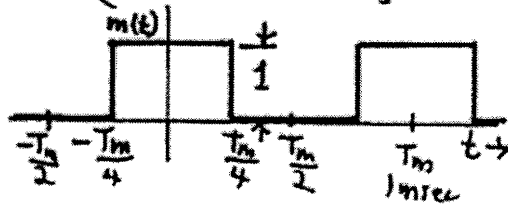
5-42

$$s(t) = \operatorname{Re}\{g(t) e^{j\omega_c t}\} = \operatorname{Re}\{10 e^{j\theta(t)} e^{j\omega_c t}\}$$

$$\theta(t) = \beta m(t)$$

$$\beta = 45^\circ \left(\frac{\pi \text{ rad}}{180^\circ} \right) = 0.785 = \pi/4$$

$$g(t) = \sum_{-\infty}^{\infty} C_n e^{jn\omega_m t}$$



$$C_n = \frac{1}{T_m} \int_{-T_m/2}^{T_m/2} g(t) e^{-jn\omega_m t} dt = \frac{10}{T_m} \left[\int_{-T_m/4}^{T_m/4} e^{j0} e^{-jn\omega_m t} dt + \int_{T_m/4}^{T_m/2} e^{j\beta} e^{-jn\omega_m t} dt + \int_{T_m/4}^{T_m/2} e^{j0} e^{-jn\omega_m t} dt \right]$$

$$C_n = \frac{10}{T_m} \left[\int_{T_m/4}^{T_m/2} e^{-jn\omega_m t} dt + \int_{-T_m/4}^{T_m/4} e^{-jn\omega_m t} dt + \int_{T_m/4}^{T_m/2} e^{-jn\omega_m t} dt \right]$$

$$= \frac{20}{T_m} \int_{T_m/4}^{T_m/2} \left(\frac{e^{-jn\omega_m t} + e^{-jn\omega_m t}}{2} \right) dt + \frac{10e^{j\beta}}{T_m} \frac{e^{-jn\omega_m t}}{-jn\omega_m} \bigg|_{-T_m/4}^{T_m/4}$$

$$= \frac{20}{T_m} \left[\frac{\sin(n\omega_m t)}{n\omega_m} \bigg|_{T_m/4}^{T_m/2} + e^{j\beta} \frac{e^{j\pi/2} - e^{-j\pi/2}}{2jn\omega_m} \right]$$

$$= \frac{20}{T_m} \left[\frac{\sin(\pi/2) - \sin(\pi/4)}{n\omega_m} + e^{j\beta} \frac{\sin(\pi/2)}{n\omega_m} \right]$$

$$C_n = 5(e^{j\beta} - 1) \left[\frac{\sin(\frac{n\pi}{2})}{\frac{n\pi}{2}} \right], \quad \beta = \pi/4$$

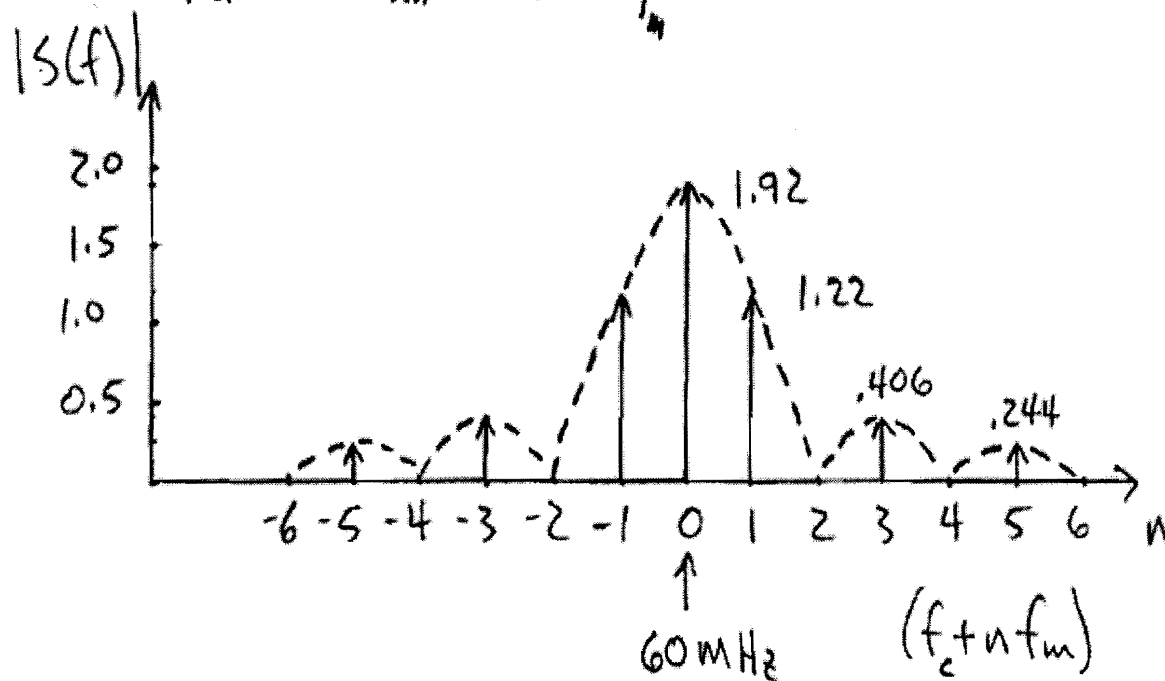
$$|C_n| = 5 \sqrt{(\cos\beta - 1)^2 + (\sin\beta)^2} \left| \frac{\sin(\frac{n\pi}{2})}{\frac{n\pi}{2}} \right|$$

$$\Rightarrow |C_n| = 3.83 \left| \frac{\sin(\frac{n\pi}{2})}{\frac{n\pi}{2}} \right|$$

$$s'(f) = \frac{1}{2} \left[\sum_{-\infty}^{\infty} C_n \delta(f - f_c - n f_m) + \sum_{-\infty}^{\infty} C_n^* \delta(f + f_c + n f_m) \right]$$

5-42 Cont'd.

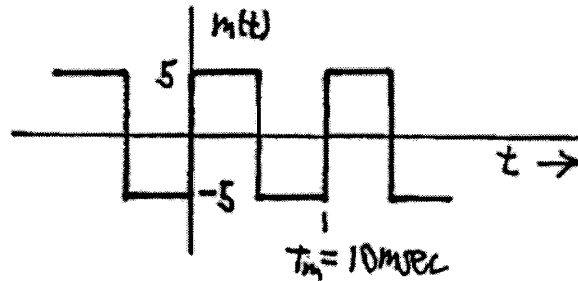
$$f_m = 1/kHz = \frac{1}{T_m}$$



5-46

NBFM

$$\theta(t) = D_f \int_0^t m(\lambda) d\lambda$$



(a.)

$$\Delta\theta = D_f \int_0^{T_m/2} m(t) dt = D_f \int_0^{T_m/2} 5 dt = D_f 5t \Big|_0^{T_m/2} = D_f 5 \frac{T_m}{2} = \underbrace{D_f 5 \frac{T_m}{2}}_{\text{Set } 1} = (10^\circ) \left(\frac{\pi \text{ rad}}{180^\circ} \right)$$

$$\Rightarrow D_f = \frac{10\pi}{180} \left(\frac{2}{5T_m} \right) = \frac{20\pi}{5(180)10^{-2}} = \underline{\underline{6.98 \frac{\text{rad}}{\text{V}\cdot\text{sec}}}}$$

$$(5-6) \Rightarrow \Delta F = \frac{D_f V_p}{2\pi} = \frac{6.98(5)}{2\pi} = \underline{\underline{5.55 \text{ Hz}}}$$

(b.) From (5-26) and (5-27)

$$S'(f) = \frac{A_c}{2} \left\{ \delta(f-f_c) + \delta(f+f_c) + \frac{D_f}{2\pi} \frac{M(f-f_c)}{f-f_c} - \frac{D_f}{2\pi} \frac{M(f+f_c)}{f+f_c} \right\}$$

$$M(f) = \mathcal{F}[m(t)] = \sum_{-\infty}^{\infty} C_n \delta(f - n f_m) \text{ where } f_m = \frac{1}{10 \text{ msec}} = \underline{\underline{100 \text{ Hz}}}$$

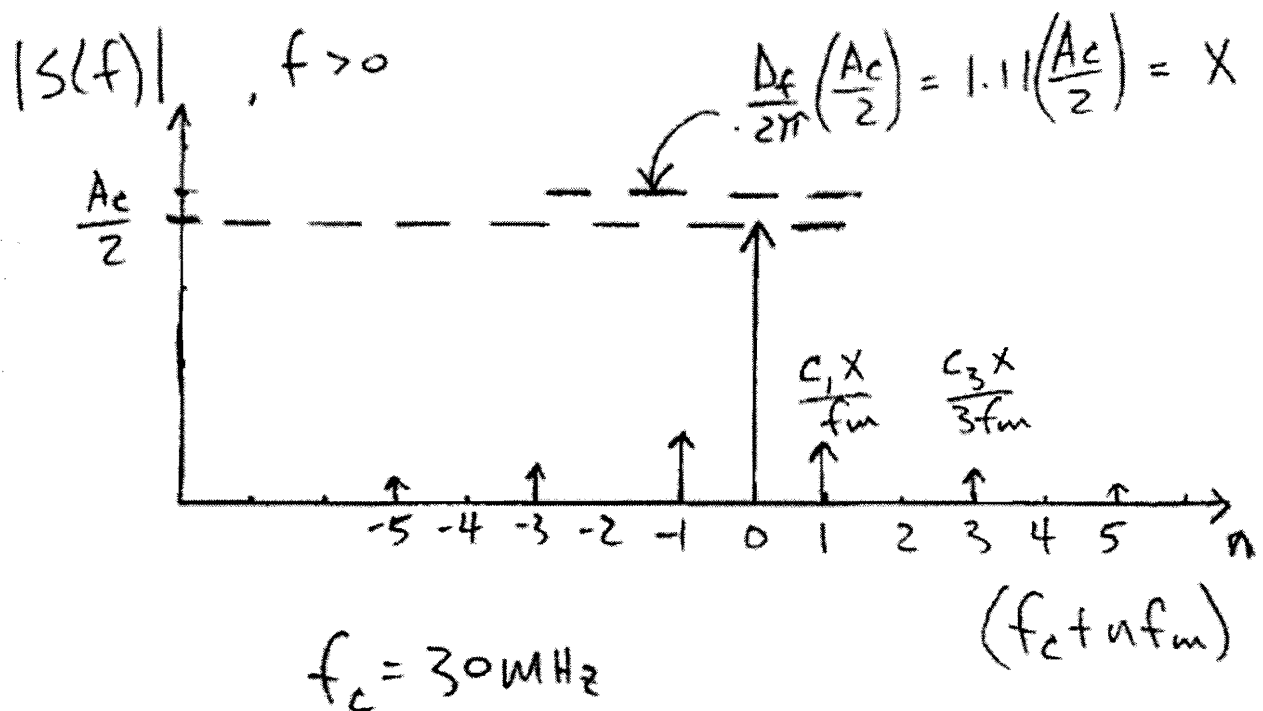
$$S'(f) = \frac{A_c}{2} \left\{ \delta(f-f_c) + \delta(f+f_c) + \frac{D_f}{2\pi} \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \left(\frac{C_n}{n f_m} \right) \left[\delta(f-f_c - n f_m) - \delta(f+f_c - n f_m) \right] \right\}$$

5-46.(b.) (cont'd. Aside : Evaluate c_n

$$\begin{aligned}
 c_n &= \frac{1}{T_m} \int_0^{T_m} m(t) e^{-jn\omega_m t} dt \\
 &= \frac{1}{T_m} \left[\int_0^{T_m/2} 5 e^{-jn\omega_m t} dt + \int_{T_m/2}^{T_m} (-5) e^{-jn\omega_m t} dt \right] \\
 &= \frac{5}{T_m} \left[\frac{e^{-jn\omega_m t}}{-jn\omega_m} \Big|_0^{T_m/2} - \frac{e^{-jn\omega_m t}}{-jn\omega_m} \Big|_{T_m/2}^{T_m} \right] \\
 &= \frac{5}{T_m} \left[\frac{e^{-jn\omega_m T_m/2} - e^{-j0} - e^{-jn\omega_m T_m} + e^{-jn\omega_m T_m/2}}{-jn\omega_m} \right] \\
 &= \frac{5}{T_m} \left[\frac{2 e^{-jn\pi} - 1 - e^{-jn2\pi}}{-jn\omega_m} \right] \\
 &= 10 \left[\frac{1 - e^{-jn\pi}}{jn\omega_m T_m} \right] = 10 e^{-jn\pi/2} \left[\frac{e^{jn\pi/2} - e^{-jn\pi/2}}{j2\pi n} \right] \\
 &= \frac{10}{2} e^{-jn\pi/2} \left[\frac{\sin(n\pi/2)}{n\pi/2} \right] \\
 c_n &= 5 (-j)^n \left[\frac{\sin n\pi/2}{n\pi/2} \right], n \neq 0; c_0 = 0
 \end{aligned}$$

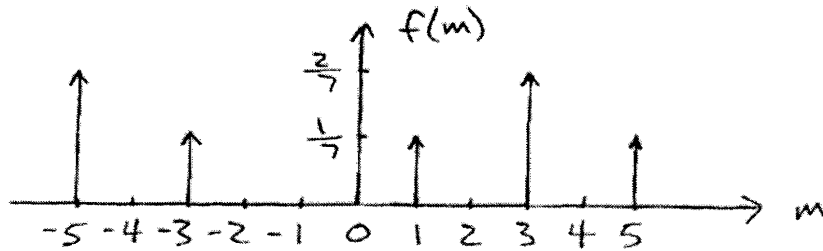
5-46. (b.) Cont'd

$$\underline{c_n = 0, n = \text{even}}$$



5-49

$$A_c = 5, f_c = 2 \text{ GHz}, \Delta f = 10^5$$



$$(4-131) \Rightarrow P(f) = \frac{\pi A_c^2}{2 \Delta f} \left[f\left(\frac{2\pi}{\Delta f}(f-f_c)\right) + f\left(\frac{2\pi}{\Delta f}(-f-f_c)\right) \right]$$

$$\delta(ax) = \frac{1}{|a|} \delta(x), \quad a = \frac{2\pi}{\Delta f}; \quad \delta(-x) = \delta(x)$$

$$= \frac{A_c^2}{4} \left\{ \frac{2}{7} \delta(f-f_c+5f_0) + \frac{1}{7} \delta(f-f_c+3f_0) \right.$$

$$+ \frac{1}{7} \delta(f-f_c-f_0) + \frac{2}{7} \delta(f-f_c-3f_0)$$

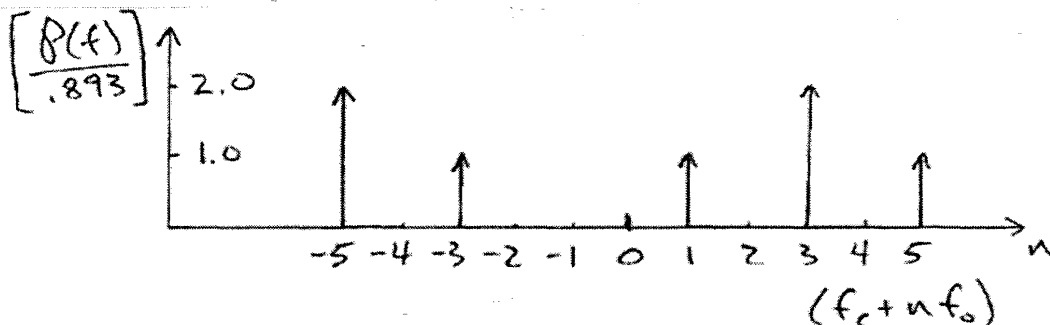
$$+ \frac{1}{7} \delta(f-f_c-5f_0) + \frac{2}{7} \delta(f+f_c-5f_0)$$

$$+ \frac{1}{7} \delta(f+f_c-3f_0) + \frac{1}{7} \delta(f+f_c+f_0)$$

$$+ \frac{2}{7} \delta(f+f_c+3f_0) + \frac{1}{7} \delta(f+f_c+5f_0) \left. \right\}$$

$$\text{where } f_0 = \frac{\Delta f}{2\pi} = 15.9 \text{ kHz}$$

$$\frac{A_c^2}{4(7)} = \frac{25}{28} = 0.893$$



5-52

(a.) Assuming the BW's are absolute BW's :

The BW of $s(t)$ is $2B_2 = \underline{\underline{20 \text{ KHz}}}$

$$\begin{aligned} s(t) &= m_1(t) \cos \omega_c t + m_2(t) \sin \omega_c t \\ &= \text{Re} \left\{ [m_1(t) - j m_2(t)] e^{j \omega_c t} \right\} \end{aligned}$$

$$g(t) = m_1(t) - j m_2(t)$$

$$G(f) = M_1(f) - j M_2(f)$$

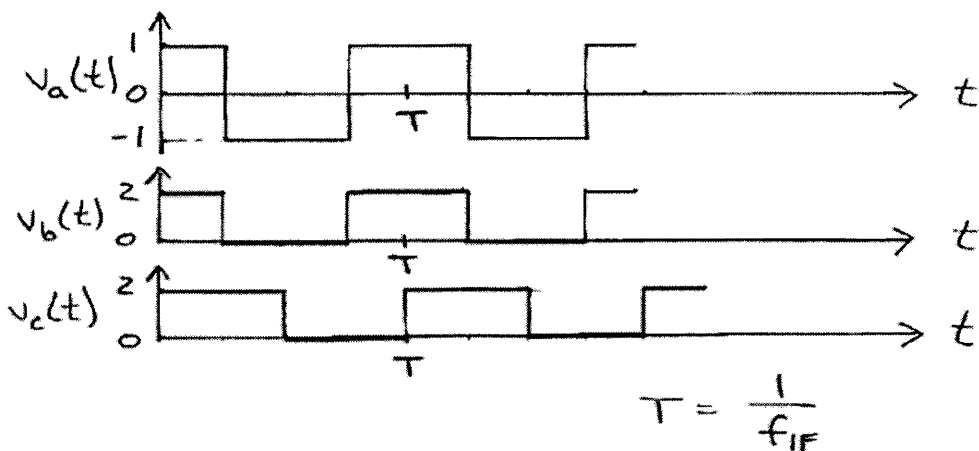
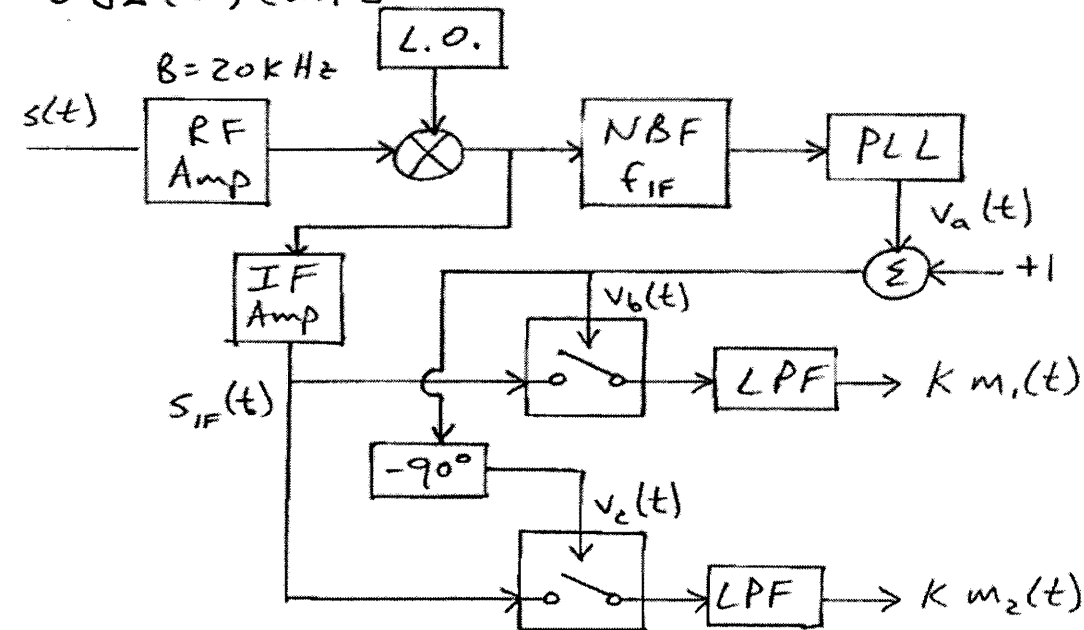
$$\begin{aligned} S(f) &= \frac{1}{2} [G(f-f_c) + G^*(-f-f_c)] \\ &= \frac{1}{2} [M_1(f-f_c) + M_1^*(-f-f_c) \\ &\quad - j M_2(f-f_c) + j M_2^*(-f-f_c)] \end{aligned}$$

for $m(t)$ real : $M^*(-f) = M(f)$

$$\begin{aligned} &= \frac{1}{2} [M_1(f-f_c) + M_1(f+f_c)] \\ &\quad - \frac{j}{2} [M_2(f-f_c) - M_2(f+f_c)] \end{aligned}$$

(c.) Assume $m_1(t)$ has a constant D.C. value, therefore providing a discrete carrier term.

5-52 (c.) cont'd



$s_{IF}(t)$ is sampled once every T seconds
by each switch, therefore

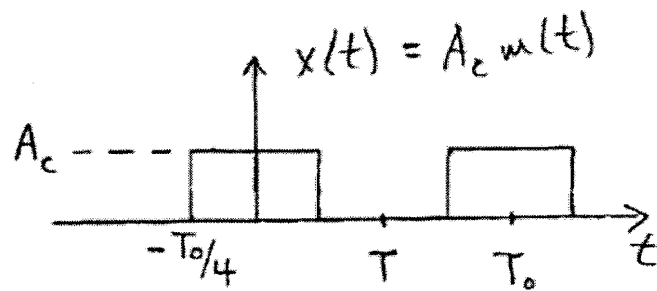
$$f_s = \frac{2}{T} = 2f_{IF}$$

$v_a(t)$ could be obtained by passing
the output of the VCO (of the PLL) through a
hard limiter.

5-53 (a.) $s(t) = x(t) \cos \omega_c t$, where

$$T = \frac{1}{R} = \frac{1}{24000}$$

$$T_0 = 2T$$



OOK :

$$S(f) = \frac{1}{2} [X(f-f_c) + X^*(-f-f_c)]$$

$$X(f) = \sum_{-\infty}^{\infty} c_n \delta(f - n f_0); \quad x(t) = \sum_{-\infty}^{\infty} c_n e^{j n \omega_0 t}$$

$$c_n = \frac{1}{T_0} \int_{-T_0/4}^{T_0/4} A_c e^{-j n \omega_0 t} dt = \frac{A_c}{T_0} \left. \frac{e^{-j n \omega_0 t}}{-j n \omega_0} \right|_{-T_0/4}^{T_0/4}$$

$$= \frac{A_c}{T_0} \frac{e^{-j n \pi/2} - e^{j n \pi/2}}{-j n 2\pi/T_0} = \frac{A_c}{2} \frac{\sin(n\pi/2)}{n\pi/2}$$

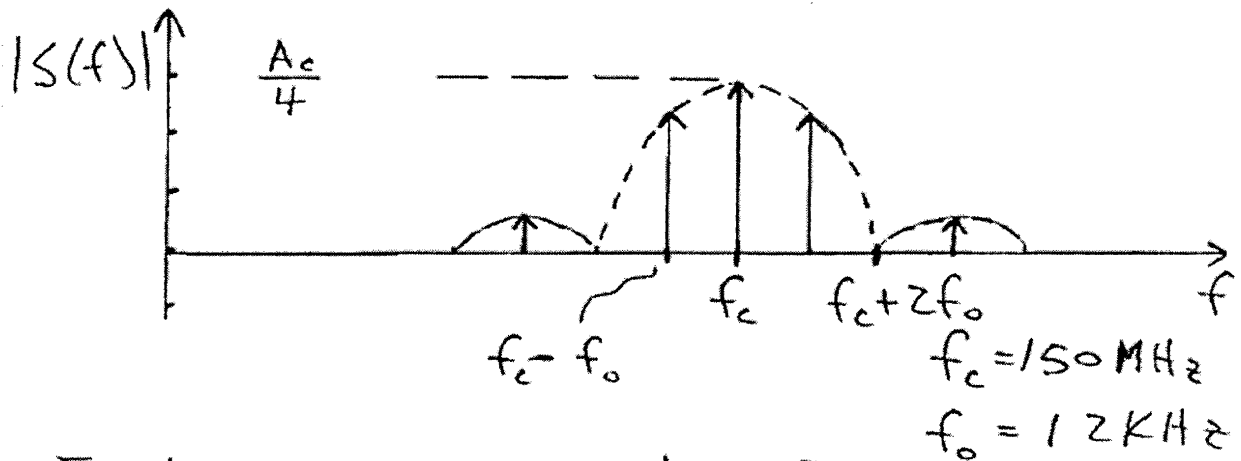
$$X(f) = \frac{A_c}{2} \sum_{-\infty}^{\infty} \left[\frac{\sin(n\pi/2)}{n\pi/2} \right] \delta(f - n f_0)$$

$$f_0 = \frac{1}{T_0} = \frac{1}{2T} = \frac{R}{2}$$

$$S(f) = \frac{1}{2} [X(f-f_c) + X^*(-f-f_c)]$$

5-53 Cont'd.

(b)



First zero-crossing at $n=2$

$$BW_T = 2(2f_0) = 2R = \underline{\underline{48 \text{ KHz}}}$$

5-57

k := 0 .. 7

a :=

1
-1
-1
1
-1
1
1
-1

T := 0.0025

t := 0, 0.00008 .. 8 T

$\omega_c := 1000 \pi$

$\text{rect}(t) := \text{if} \left[\left| t - \frac{T}{2} \right| \leq \frac{T}{2}, 1, 0 \right]$

$m(t) := \sum_k [a_k \text{rect}(t - k T)]$

SET INDEX
(a) h := 0.2

N := 64

n := 1 .. N - 1

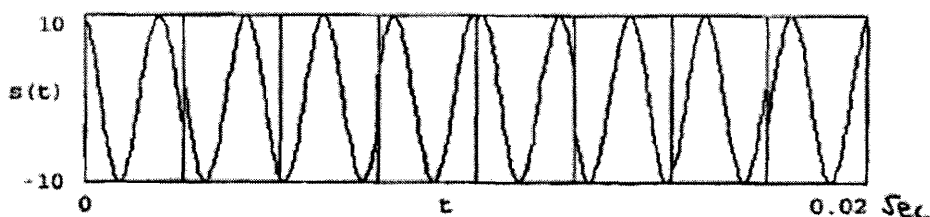
j := 0 .. $\frac{N}{2}$

$D_p := \frac{h \pi}{2}$

$T_s := \frac{0.02}{N}$

$t1_n := n T_s$

$s(t) := 10 \cos \left[\omega_c t + D_p m(t) \right]$



M := 6

N := 2

N = 64

k := 0 .. N - 1

T0 := 8 T

$dt := \frac{T0}{N}$

dt = 0

$t_k := k dt$

$ss_k := s(k \cdot dt)$

n := 0 .. N - 1

$NN := \frac{M-1}{2}$

Assume that the signal is periodic with period $T0=8T$. The spectrum can be obtained from the complex Fourier series coefficients. Furthermore, the complex Fourier series coefficients may be calculated using the FFT by substituting (2-178) into (2-186). Thus,

$c_n := \frac{1}{\sqrt{N}} \text{icfft}(ss)$

$fs := \frac{1}{dt}$

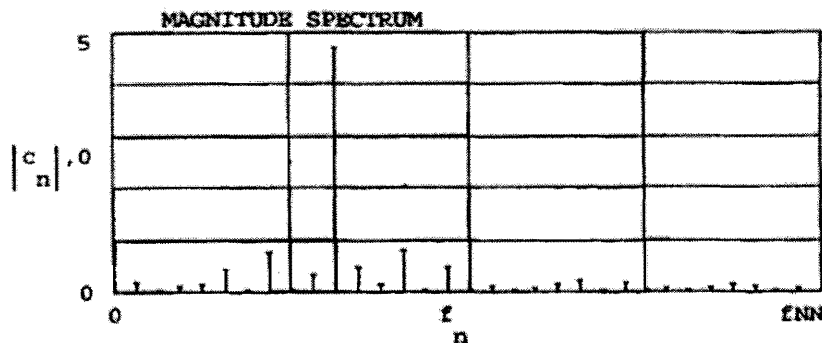
fs = 3200 Hz

$f_n := \frac{n}{T0}$

$f_1 = 50$

$f_{NN} := NN f_1$

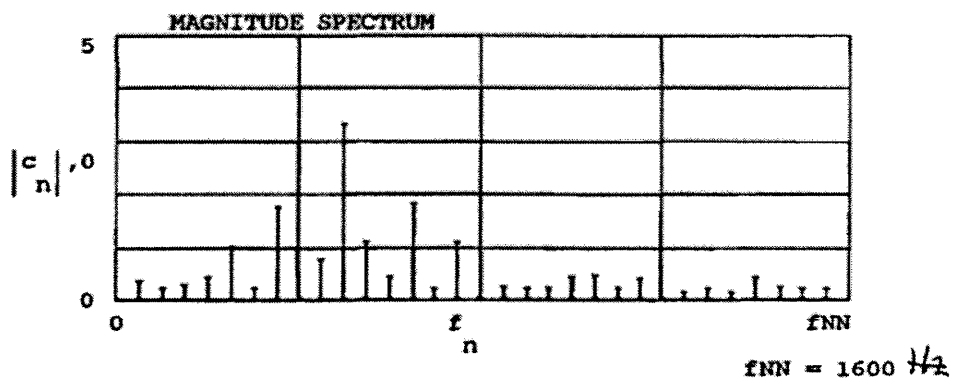
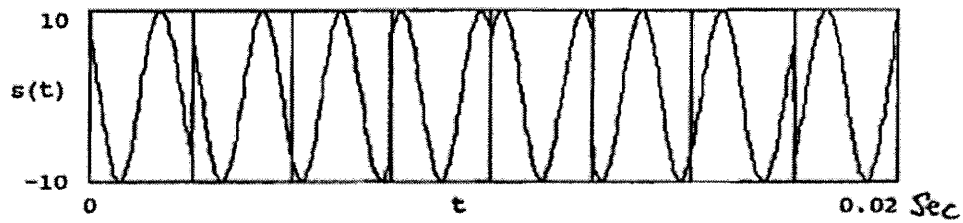
$f_{NN} = 1600$



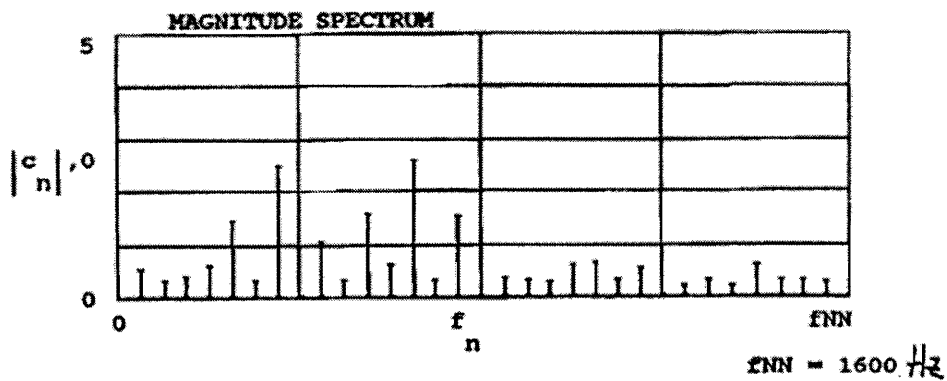
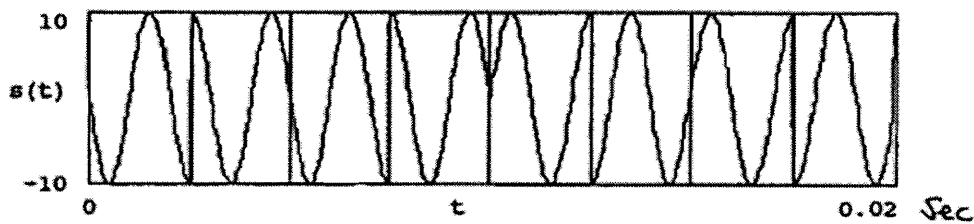
$f_{NN} = 1600 \text{ Hz}$

5-57 Cont'd

(b) $h = 0.5$



(c) $h = 1.0$



5-62

$$\Delta = R = \frac{2B}{1+r}$$

(a.) OOK

$$(5-2) \quad B_{\eta} = 2B = (1+r)R = (1.5)64k \\ = \underline{\underline{96 \text{ kHz}}}$$

(b.) FSK

$$(5-7) \quad B_{\eta} = 2\Delta F + 2B \\ = 5 \text{ kHz} + 96 \text{ kHz} = \underline{\underline{101 \text{ kHz}}}$$

5-67

Use (5-106) $B_{\eta} = (1+r)\frac{B}{l}$ where $l=2$ for QPSK

$$(a.) \Rightarrow 24 = (1+r)\frac{30}{2} \Rightarrow (1+r) = \frac{2(24)}{30} = 1.6 \\ \text{or } \underline{\underline{r = 0.6}}$$

(b.) Max R allowed is when $r=0$

$$\Rightarrow R_{\max} = \frac{2B_{\eta}}{1} = 2(24) = 48 \text{ Mb/sec}$$

No. A roll-off factor, r , could not be found support 50 Mb/s QPSK signaling

5-68

(a) Using the same procedure that leads to (5-102) where the FT of the square-root raised-cosine-rolloff pulse is obtained from (3-69)

$$F(f) = \sqrt{H_e(f)}$$

Then, using (6-70d) with $m_c = 0$ and $v_c^2 = C$, the PSD of the complex envelope is

$$P_g(f) = K |F(f)|^2 = K H_e(f)$$

$$\neq P_g(f) = K \begin{cases} 1, & |f| < f_1 \\ \frac{1}{2} \left[1 + \cos \left[\frac{\pi (|f| - f_1)}{2 f_a} \right] \right], & f_1 < |f| < B \\ 0, & |f| > B \end{cases}$$

where $f_0 = \frac{1}{2T_b} = \frac{1}{2} \left(\frac{R}{T_b} \right)$ and $\ell = 2$ for QPSK

$R = \frac{1}{T_b}$

5-68 Continued

$$f_1 = f_0 - f_{\Delta} = f_0 - r f_0 = f_0 (1-r) = \frac{1}{2} (1-r) \left(\frac{R}{l}\right)$$

$$f_{\Delta} = r f_0 = \frac{1}{2} r \left(\frac{R}{l}\right)$$

and

$$B = f_0 + f_{\Delta} = f_0 (1+r) = \frac{1}{2} (1+r) \left(\frac{R}{l}\right)$$

Absolute BW
of complex
envelope

$$P_{cr}(f) = \frac{1}{4} [P_q(f-f_c) + P_q(f+f_c)]$$

$$P_q(f) = P_q(f) \text{ for QPSK} \rightarrow P_q(f \pm f_c)$$

The absolute bandpass bandwidth is

$$B_{\text{TP}} = 2B = (1+r) \left(\frac{R}{l}\right) \text{ where } l=2 \text{ for QPSK}$$

(b.)

$r := 0.35$ ← Enter value of the rolloff factor, r

$R := 1$ $f_0 := 0.5 \cdot \left(\frac{R}{2}\right)$ $N := 100$ $n := 0, 1, \dots, N-1$

$df := 2 \cdot \frac{f_0}{(N-1)}$ $f_n := n \cdot df$

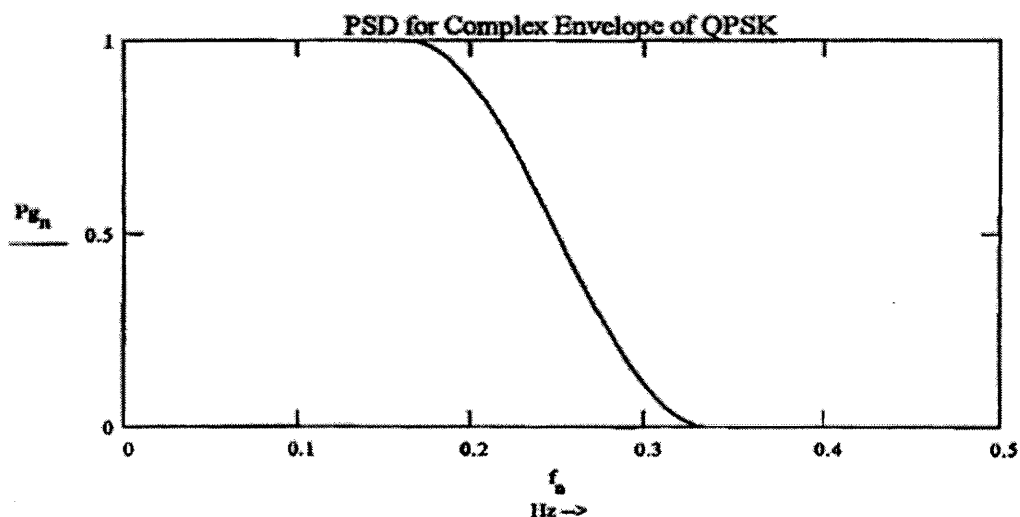
$f_{\text{del}} := r \cdot f_0$ $f_1 := f_0 - f_{\text{del}}$ $B := f_0 + f_{\text{del}}$

Construct the Raised Cosine frequency response using the Mathcad "if" function.

$$\text{cos1}(f) := 0.5 \left[1 + \cos \left[0.5 \cdot \pi \cdot \frac{(f-f_1)}{f_{\text{del}}} \right] \right]$$

$$P_{g1}(f) := \text{if}(f > f_1, \text{cos1}(f), 1) \quad P_g(f) := \text{if}(f > B, 0, P_{g1}(f))$$

$$P_{g_n} := P_g(n \cdot df)$$



5-77

From the description of $\pi/4$ QPSK in Sec. 5-10, use the table shown at the right

Input Bits	$\Delta\theta$
11	$+45^\circ$
01	$+135^\circ$
00	-135°
10	-45°

Data	10	11	01	00	10	10	10
$\Delta\theta$	-45°	$+45^\circ$	$+135^\circ$	-135°	-45°	-45°	-45°

5-82

```

i := 0 .. 14    n := 0 .. 14    t := -5, -4.99 .. 5    T := 1    a := -1
a1 := -1    a2 := -1    a3 := 1    a4 := 1    a5 := 1    a0 := -1
a7 := -1    a8 := -1    a9 := 1    a10 := -1    a11 := -1    a12 := 1
a13 := -1    a14 := -1
tt := 0, 0.2 .. 15
h(t) :=  $\phi(t) - \phi(t - 1)$     n1 := 0, 2 .. 14    n2 := 1, 3 .. 13

```

5-82 Cont'd.

$$h_x(t) := \frac{1}{2}(t) - \frac{1}{2}(t - 2)$$

$$m(tt) := \sum_n a_n \cdot h(tt - nT)$$

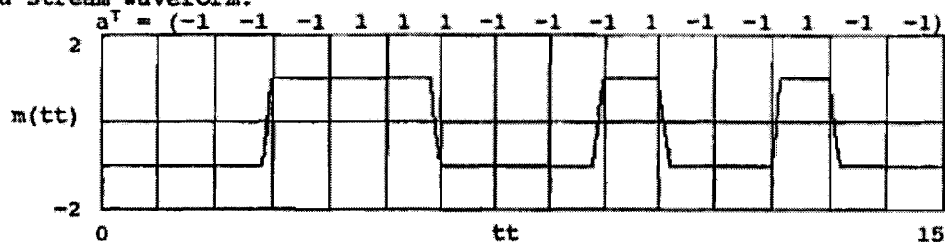
$$x(tt) := \sum_{n1} a_{n1} \cdot h_x(tt - n1T)$$

$$y(tt) := \sum_{n2} a_{n2} \cdot h_x(tt - n2T)$$

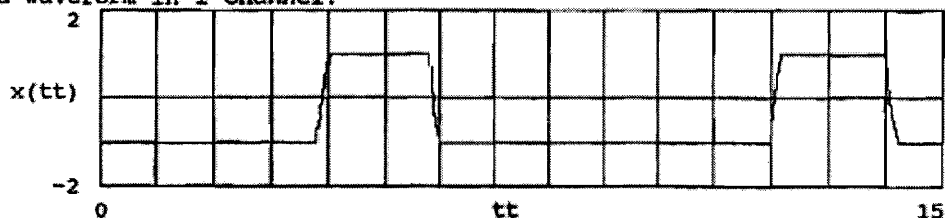
$$yy(tt) := \sum_{n2} a_{n2} \cdot h_x(tt - n2T) \cdot \sin\left[\pi \frac{tt - n2T}{2}\right]$$

$$xx(tt) := \sum_{n1} a_{n1} \cdot h_x(tt - n1T) \cdot \cos\left[\pi \frac{tt - n1T}{2}\right]$$

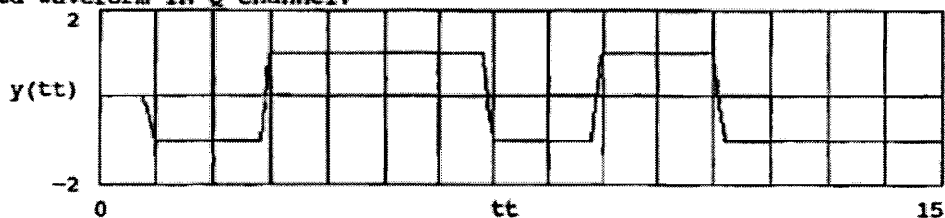
Data stream waveform:



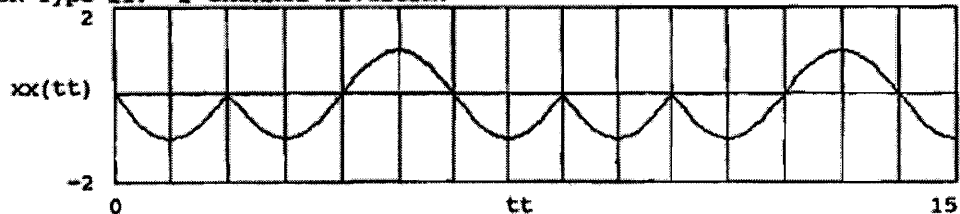
Data waveform in I-channel:



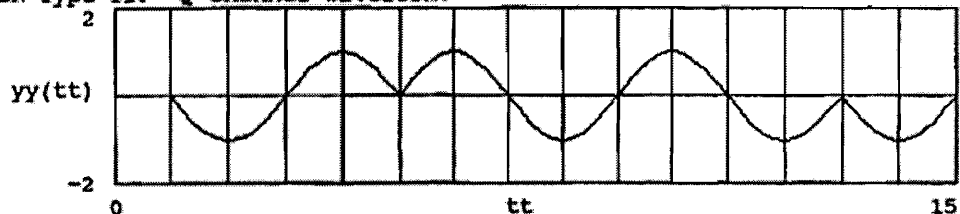
Data waveform in Q-channel:



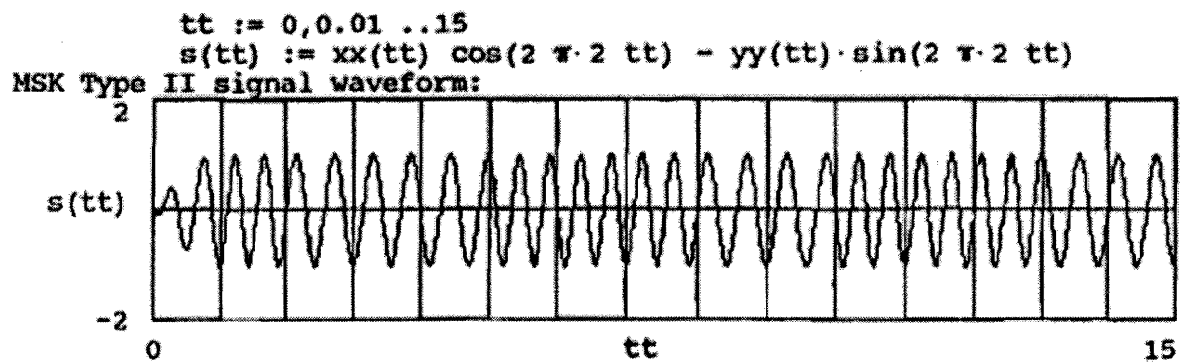
MSK Type II: I-channel waveform:



MSK Type II: Q-channel waveform:



5-82 Cont'd.



5-84 (a) $H(f) = e^{-\left(\frac{f}{B}\right)^2 \frac{\ln 2}{2}} = \left[e^{-\pi \left(f \sqrt{\frac{\ln 2}{2}} \frac{1}{B} \frac{1}{\pi}\right)^2} \right] \Rightarrow h(t) = \frac{B \sqrt{2\pi}}{\sqrt{\ln 2}} e^{-\pi \left(t B \sqrt{\frac{2\pi}{\ln 2}}\right)^2}$

$\pi\left(\frac{t}{T}\right) \rightarrow h(t) \rightarrow h_s(t)$ $\pi(-t) = \pi(t)$ \uparrow Table 2-2

$h_s(t) = \pi\left(\frac{t}{T}\right) * h(t) = \int h(\lambda) \pi\left(\frac{t-\lambda}{T}\right) d\lambda \stackrel{?}{=} \int h(\lambda) \pi\left(\frac{\lambda-t}{T}\right) d\lambda$

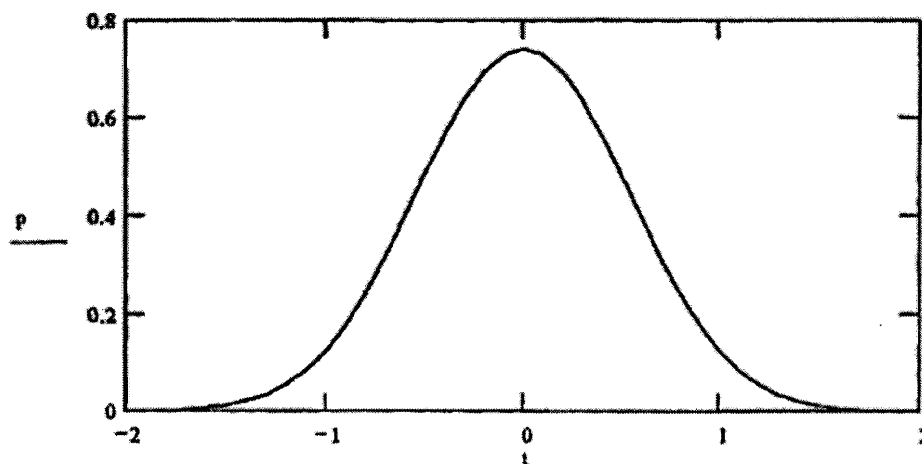
$= \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \left(\frac{\sqrt{2\pi}}{\sqrt{\ln 2}}\right) B e^{-\frac{2\pi^2}{\ln 2} \lambda^2 B^2} 1 d\lambda$

$\Rightarrow h_s(t) = \left(\frac{\sqrt{2\pi}}{\sqrt{\ln 2}}\right) (BT) \int_{\frac{t}{BT} - \frac{1}{2}}^{\frac{t}{BT} + \frac{1}{2}} e^{-\frac{2\pi^2}{\ln 2} (BT)^2 x^2} dx = p(t)$

Let $\lambda = T x$
 $d\lambda = T dx$
 $x = \frac{\lambda}{T}$

(b) $BTb := 0.3 \quad Tb := 1 \quad N := 20 \quad dt := \frac{2 \cdot Tb}{N} \quad n := 0, 1, 2, \dots, N \quad t_n := (n - N) \cdot dt$

$p(t) := \left[\sqrt{2 \cdot \frac{\pi}{\ln(2)}} \cdot BTb \cdot \int_{\frac{t}{Tb} - 0.5}^{\left(\frac{t}{Tb}\right) + 0.5} e^{-\left[(2) \cdot \frac{(\pi^2) \cdot (BTb^2) \cdot x^2}{\ln(2)} \right]} dx \right] \quad p_n := p(t_n)$



5-90

(a) Referring to Fig 5-42a, the FSK signal is

$$v_1(t) = \cos[\omega_c t + \theta(t)] \text{ where } \theta(t) = D_f \int_0^t m(\lambda) d\lambda$$

The output of the FH spreader is

$$\begin{aligned} v_2(t) &= A_c \cos[\omega_c t + \theta(t)] \cos(\omega_i t) \\ &= \frac{A_c}{2} \cos[(\omega_c - \omega_i)t + \theta(t)] + \frac{A_c}{2} \cos[(\omega_c + \omega_i)t + \theta(t)] \end{aligned}$$

The output of the BPF is the sum frequency part of $v_2(t)$

$$\Rightarrow s(t) = \frac{A_c}{2} \cos[(\omega_c + \omega_i)t + \theta(t)]$$

(b) Referring to Fig 5-42b the signal out of the FH spreader

$$\begin{aligned} \text{is } v_3(t) &= s(t) 2 \cos(\omega_i t) = A_c \cos[(\omega_c + \omega_i)t + \theta(t)] \cos(\omega_i t) \\ &= \underbrace{\frac{A_c}{2} \cos[\omega_c t + \theta(t)]}_{\text{diff term}} + \underbrace{\frac{A_c}{2} \cos[(\omega_c + 2\omega_i)t + \theta(t)]}_{\text{sum term}} \end{aligned}$$

\Rightarrow The output of the BPF is $v_4(t) = \frac{A_c}{2} \cos[\omega_c t + \theta(t)]$ which is FSK.