

Chapter 6

6-2

(a.)

$$\overline{x(t)} = \int_0^{\pi/2} \frac{2}{\pi} \cdot A \cos(\omega_0 t + \theta) d\theta$$

$$= \frac{2A}{\pi} \sin(\omega_0 t + \theta) \Big|_0^{\pi/2}$$

$$= \frac{2A}{\pi} \left[\sin(\omega_0 t + \pi/2) - \sin \omega_0 t \right]$$

$$= \frac{2A}{\pi} \left[\cos \omega_0 t - \sin \omega_0 t \right]$$

$$= \frac{2\sqrt{2}A}{\pi} \left[\cos\left(\frac{\pi}{4}\right) \cos \omega_0 t - \sin\left(\frac{\pi}{4}\right) \sin \omega_0 t \right]$$

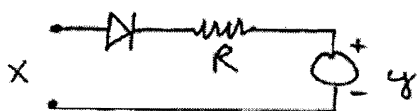
$$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} = \sin \frac{\pi}{4}$$

$$= \underline{\underline{\frac{2\sqrt{2}A}{\pi} \cos\left(\omega_0 t + \frac{\pi}{4}\right)}}$$

(b.) $\overline{x(t)}$ is a function of time

$\therefore x(t)$ is not stationary.

6-5



Let y be the current thru the meter produced by the voltage x .

For $x = V_p \cos \omega_0 t$:

$$\begin{aligned} \langle y \rangle &= \frac{1}{T_0} \int_{-T_0/4}^{T_0/4} \frac{V_p}{R} \cos \omega_0 t \, dt = \frac{2V_p}{T_0 R} \int_0^{T_0/4} \cos \omega_0 t \, dt \\ &= \frac{2V_p}{R T_0} \left. \frac{\sin \omega_0 t}{\omega_0} \right|_0^{T_0/4} = \frac{V_p}{R \pi} = \frac{\sqrt{2}}{R \pi} V_{rms} \end{aligned}$$

$$V_{meter} = V_{rms} = \frac{R \pi}{\sqrt{2}} \langle y \rangle$$

$$\omega_0 = \frac{2\pi}{T_0}$$

Let x be a zero-mean, ergodic Gaussian noise voltage.

$$\langle y \rangle = \frac{1}{R} \int_0^\infty x p(x) \, dx = \frac{1}{R \sqrt{2\pi} \Delta} \int_0^\infty x e^{-x^2/2\Delta^2} \, dx$$

where Δ = rms value of x

$$\text{Let } z = \frac{x^2}{2\Delta^2} \quad ; \quad dz = \frac{x}{\Delta^2} \, dx$$

$$\begin{aligned} \langle y \rangle &= \frac{\Delta}{R \sqrt{2\pi}} \int_0^\infty e^{-x^2/2\Delta^2} \left(\frac{x}{\Delta^2} \right) \, dx \\ &= \frac{\Delta}{R \sqrt{2\pi}} \int_0^\infty e^{-z} \, dz = \frac{-\Delta e^{-z}}{R \sqrt{2\pi}} \Big|_0^\infty \end{aligned}$$

$$= \frac{-\Delta}{R \sqrt{2\pi}} [e^{-\infty} - e^{-0}] = \frac{\Delta}{R \sqrt{2\pi}} = \langle y \rangle$$

$$V_{meter} = \frac{R \pi}{\sqrt{2}} \langle y \rangle = \frac{R \pi}{\sqrt{2}} \left[\frac{\Delta}{R \sqrt{2\pi}} \right] = \frac{\sqrt{\pi}}{2} \Delta$$

$$\therefore \underline{\underline{\Delta = V_{meter} \left[\frac{2}{\sqrt{\pi}} \right]}}$$

6-9Ergodicity $\Rightarrow \langle \{ \} \rangle = \overline{[\]}$

$$\begin{aligned}
 (a.) \quad P &= \overline{n^2(t)} = \overline{[n_1(t) + n_2(t)]^2} \\
 &= \overline{n_1^2(t)} + 2 \overline{n_1(t) n_2(t)} + \overline{n_2^2(t)} \\
 &= \overline{n_1^2(t)} + \overline{n_2^2(t)} \quad \text{orthogonal} \\
 &= 5 + 10 = \underline{\underline{15 \text{ Watts}}}
 \end{aligned}$$

$$\begin{aligned}
 (b.) \quad P &= \overline{n^2(t)} = \overline{n_1^2(t)} + 2 \overline{n_1(t) n_2(t)} + \overline{n_2^2(t)} \\
 &\quad \text{uncorrelated} \quad 2 \overline{n_1(t) n_2(t)}
 \end{aligned}$$

$$P = 5 + 2(-2)(1) + 10 = \underline{\underline{11 \text{ Watts}}}$$

$$\begin{aligned}
 (c.) \quad P &= \overline{n^2(t)} = \overline{[n_1(t) + n_2(t)]^2} \\
 &= \overline{n_1^2(t)} + 2 \overline{n_1(t) n_2(t)} + \overline{n_2^2(t)} \\
 &\quad 2 R_{n_1, n_2}(0) \\
 &= 5 + 2(2) + 10 = \underline{\underline{19 \text{ Watts}}}
 \end{aligned}$$

6-11

(a.) $\sin \omega_0 \tau$ ① $R(\tau) \neq R(-\tau)$ x
No

(b.) $\frac{(\sin \omega_0 \tau)}{\omega_0 \tau}$ ① $R(\tau) = R(-\tau)$

② $R(0) \geq |R(\tau)|$

YES

③ $\mathcal{F}\left\{\frac{\sin \omega_0 \tau}{\omega_0 \tau}\right\} = \text{Non-negative rectangle}$

(c.) $\cos \omega_0 \tau + \delta(\tau)$ ① $R(\tau) = R(-\tau)$

YES

② $R(0) \geq |R(\tau)|$

③ $\mathcal{F}\left\{\cos \omega_0 \tau + \delta(\tau)\right\} = \frac{1}{2} \delta(f-f_0) + \frac{1}{2} \delta(f+f_0) + 1$
 ≥ 0

(d.) $e^{-a/|\tau|}$ where a < 0 ① $R(\tau) = R(-\tau)$

No

② $R(0) \neq |R(\tau)|$ x

$$R_x(z) = 4e^{-z^2} + 3$$

$$\Rightarrow \mathcal{P}_x(f) \stackrel{\text{Using Table 2-2}}{=} \frac{4\sqrt{\pi}}{\pi} e^{-\pi(f\sqrt{\pi})^2} + 3\delta(f) = \underline{\underline{\frac{4\sqrt{\pi}}{\pi} e^{-\pi(f\sqrt{\pi})^2} + 3\delta(f)}}$$

(b.) This is a low-pass spectrum. \Rightarrow Use (6-97) and (6-98).

$$\overline{f_2} = \frac{\int_{-\infty}^{\infty} f_2(x) dx}{\int_{-\infty}^{\infty} p_x(x) dx} = \frac{4 \int_{-\infty}^{\infty} x^2 \frac{1}{\sqrt{2\pi}} e^{-x^2/2(\frac{1}{2\pi})} dx}{4 \left(\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2(\frac{1}{2\pi})} dx + \int_{-\infty}^{\infty} \delta(x) dx \right)}$$

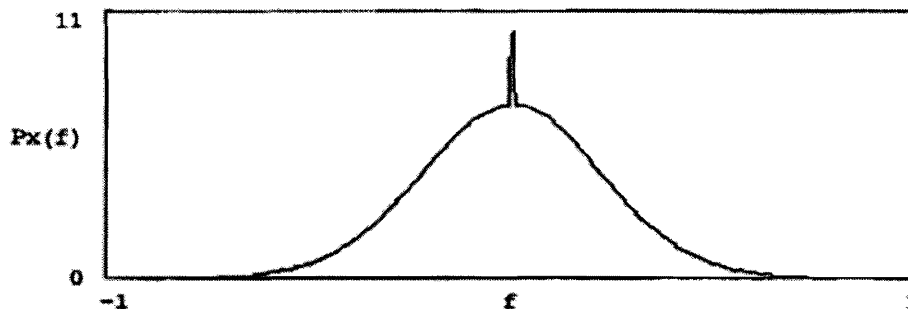
$$\Rightarrow \overline{f^2} = \frac{4 \left(\frac{1}{\sqrt{2} \pi} \right)^2}{4 + 3} = \frac{4}{7} \frac{1}{2\pi^2} = \frac{2}{7\pi^2}$$

Thus, $B_{rms} = \sqrt{f^2} = \sqrt{\frac{2}{7\pi^2}} = \sqrt{\frac{2}{7}} \frac{1}{\pi} = \underline{\underline{0.170 \text{ Hz}}}$

```
f := -1,-0.99 .. 1
```

$$\S(f) := \text{if}(f \approx 0, 1, 0)$$

$$P_X(f) := 4 \sqrt{\pi} \cdot e^{-\left[(\pi \cdot f)^2\right]} + 3 \cdot \delta(f) \quad P_X(0) = 10.09$$



$$\text{Bras} := \sqrt{\frac{2}{7}} \begin{bmatrix} 1 \\ - \\ \pi \end{bmatrix} \quad \text{Bras} = 0.17$$

6-18

$$(a.) \quad x_{rms} = \sqrt{\overline{x^2(t)}}$$

$$\overline{x^2(t)} = R_x(0) = \int_{-\infty}^{\infty} P_x(f) df$$

$$= \int_{-B}^0 \frac{1}{B} (B+f) df + \int_0^B \frac{1}{B} (B-f) df$$

$$= \frac{1}{B} \left[\left(Bf + \frac{f^2}{2} \right) \Big|_{-B}^0 + \left(Bf - \frac{f^2}{2} \right) \Big|_0^B \right]$$

$$= \frac{1}{B} \left[- \left(-B^2 + \frac{B^2}{2} \right) + \left(B^2 - \frac{B^2}{2} \right) \right] = \frac{1}{B} [2B^2 - B^2]$$

$$= B \Rightarrow \underline{\underline{x_{rms} = \sqrt{B}}}$$

$$(b.) \quad P_x(f) = \frac{1}{\sqrt{B}} \pi\left(\frac{f}{B}\right) * \frac{1}{\sqrt{B}} \pi\left(\frac{f}{B}\right)$$

$$R_x(\tau) = \mathcal{F}^{-1}[P_x(f)] = \frac{1}{B} \left\{ \mathcal{F}^{-1}\left[\pi\left(\frac{f}{B}\right)\right] \right\}^2$$

Table 2-1 · Multiplication property of $\mathcal{F}\{\}$

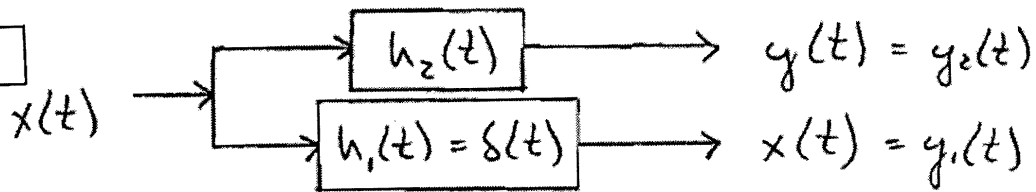
$$R_x(\tau) = \frac{1}{B} \left\{ \mathcal{F}^{-1}\left[\pi\left(\frac{f}{B}\right)\right] \right\}^2$$

Table 2-2

$$\Rightarrow = \frac{1}{B} \left\{ B \frac{\sin(\pi B \tau)}{\pi B \tau} \right\}^2$$

$$\underline{\underline{R_x(\tau) = B \left[\frac{\sin(\pi B \tau)}{\pi B \tau} \right]^2}}$$

6-22



$$P_x(f) = \frac{N_0}{2} \Rightarrow R_x(\tau) = \frac{N_0}{2} \delta(\tau)$$

(a.) From eqn. (6-86 b.)

$$R_{y,y_2}(\tau) = h_1(-\tau) * h_2(\tau) * R_{x,x_2}(\tau)$$

$$\text{but } h_1(-\tau) = \delta(-\tau) = \delta(\tau)$$

$$\begin{aligned} \Rightarrow R_{xy}(\tau) &= \delta(\tau) * h_2(\tau) * R_x(\tau) \\ &= \delta(\tau) * \left[h_2(\tau) * \frac{N_0}{2} \delta(\tau) \right] \\ &= \delta(\tau) * \frac{N_0}{2} h_2(\tau) = \frac{N_0}{2} h_2(\tau) \end{aligned}$$

$$\Rightarrow \underline{\underline{h_2(\tau) = R_{xy}(\tau) \left[\frac{2}{N_0} \right]}}$$

$$(b.) \text{ From (a.) : } R_{xy}(\tau) = \frac{N_0}{2} h_2(\tau)$$

$$P_{xy}(f) = \mathcal{F}[R_{xy}(\tau)] = \mathcal{F}\left[\frac{N_0}{2} h_2(\tau)\right] = \frac{N_0}{2} H_2(f)$$

$$\Rightarrow \underline{\underline{H_2(f) = \left[\frac{2}{N_0} \right] P_{xy}(f)}}$$

6-25

$$(a.) P_y(f) = |H(f)|^2 P_n(f) = \left| \frac{K}{j2\pi f} \right|^2 \frac{N_0}{2}$$

$$\underline{P_y(f) = \frac{N_0 K^2}{8\pi^2 f^2}}$$

$$(b.) y_{rms}^2 = R_y(0) = \int_{-\infty}^{\infty} P_y(f) df$$

$$= \int_{-\infty}^{\infty} \frac{N_0 K^2}{8\pi^2 f^2} df = 2 \int_0^{\infty} \left[\frac{N_0 K^2}{8\pi^2} \right] \frac{1}{f^2} df$$

$$= \frac{N_0 K^2}{4\pi^2} \left[\frac{-1}{f} \right]_0^{\infty} = \frac{N_0 K^2}{4\pi^2} \left[\frac{-1}{\infty} + \frac{1}{0} \right] = \underline{\underline{\infty}}$$

A practical integrator will have a large (i.e. finite) output.

6-29

$$\text{From (6-95): } \left(\frac{S}{N} \right)_{out} = \frac{2A_0^2 RC}{N_0 [1 + (2\pi f_0 RC)^2]}$$

$$\text{Let } RC = z, \quad 2\pi f_0 = \omega_0$$

$$\Rightarrow \left(\frac{S}{N} \right)_{out} = \frac{2A_0^2 z}{N_0 [1 + (\omega_0 z)^2]}$$

$$\text{For } \max \left[\left(\frac{S}{N} \right)_{out} \right], \text{ set } \frac{d \left[\left(\frac{S}{N} \right)_{out} \right]}{dz} = 0$$

$$\Rightarrow \frac{d \left[\left(\frac{S}{N} \right)_{out} \right]}{dz} = \frac{[1 + (\omega_0 z)^2] 2A_0^2 - 2A_0^2 z (\omega_0^2 z)}{N_0 [1 + (\omega_0 z)^2]^2}$$

$$\text{Set numerator} = 0 \Rightarrow 2A_0^2 [1 + \omega_0^2 z^2 - 2z\omega_0^2 z] = 0$$

$$\Rightarrow [1 - \omega_0^2 z^2] = 0 \Rightarrow \omega_0^2 z^2 = 1 \Rightarrow z^2 = \frac{1}{\omega_0^2} \Rightarrow z = \frac{1}{\omega_0}$$

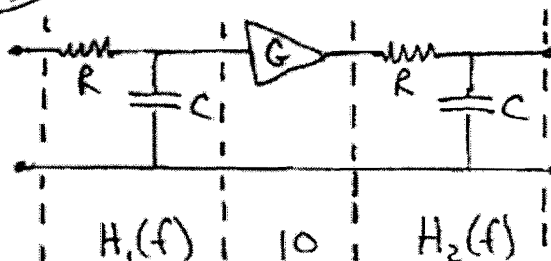
$$\text{Thus, } \underline{\underline{RC = \frac{1}{2\pi f_0}}} \text{ for } \max \left(\frac{S}{N} \right)_{out}$$

6-34

(a.)

eqn. (6-102)

$\Rightarrow f_0$



$$H_1(f) = H_2(f) = \frac{1}{1 + j f/f_0}$$

where $f_0 = \frac{1}{2\pi RC}$, and $G = 10$

$$H(f) = H_1(f)[10]H_2(f) = \frac{10}{[1 + j f/f_0]^2}$$

$$|H(f)| = \frac{10}{|1 + j 2f/f_0 - (f/f_0)^2|} = \frac{10}{\sqrt{[1 - (f/f_0)^2]^2 + (2f/f_0)^2}}$$

Using a programmable calculator, find the value of $f = f_c$, such that:

$$|H(f_c)| = \frac{10}{\sqrt{2}} \Rightarrow \underline{f_c = 0.690 f_0 ; f_0 = \frac{1}{2\pi RC}}$$

6-37

(a.) x_1 & y_2 uncorrelated \Rightarrow Independent

prop. 3

when $R_{xy}(\tau) = \overline{x(t_1)y(t_2)} = 10\sin(2\pi\tau) = 0 = \cancel{x_1} \cancel{y_2}$

$\Rightarrow 2\pi\tau = \pm n\pi \Rightarrow$ These r.v.'s are independent

only when $t_2 - t_1 = \tau = \pm \frac{n}{2}$; $n = 0, 1, \dots$

6-37 Cont'd (b.)

$$10 \sin(2\pi t) = 10 \sin[2\pi(t_2 - t_1)] = \overline{x(t_1) y(t_2)}$$

This cannot be expressed as $\overline{x(t_1) y(t_2)}$

$\therefore x(t)$ and $y(t)$ are not indep.

6-39Evaluate $\overline{x^2(t)} = R_x(0)$

$$\overline{x^2(t)} = \overline{A_o^2 \cos^2(\omega_o t + \theta)} = \frac{A_o^2}{2} \left[1 + \overline{\cos(2\omega_o t + 2\theta)} \right]$$

$$= \frac{A_o^2}{2} + \frac{A_o^2}{2} \int_0^{\pi/2} \cos(2\omega_o t + 2\theta) \frac{2}{\pi} d\theta$$

$$= \frac{A_o^2}{2} + \frac{A_o^2}{2} \frac{\sin(2\omega_o t + 2\theta) \left(\frac{2}{\pi} \right)}{2} \bigg|_0^{\pi/2}$$

$$= \frac{A_o^2}{2} + \frac{A_o^2}{2\pi} \left[\sin(2\omega_o t + \pi) - \sin(2\omega_o t) \right]$$

$$= \frac{A_o^2}{2} + \frac{A_o^2}{2\pi} \left[-2 \sin(2\omega_o t) \right] \quad \begin{array}{l} \text{This is a function} \\ \text{of } t \therefore x(t) \text{ not} \\ \text{W.S.S.} \end{array}$$

Using Sec. A-1

6-40

$$\text{Eqn. (6-42)} \quad P_x(f) = \lim_{T \rightarrow \infty} \left[\frac{|X_T(f)|^2}{T} \right]$$

$$\text{where } X_T(f) = \int_{-T/2}^{T/2} x(t) e^{-j\omega t} dt$$

$$= \int_{-T/2}^{T/2} A_0 \cos(\omega_0 t + \theta) e^{-j\omega t} dt$$

$$= A_0 \int_{-T/2}^{T/2} \frac{e^{j(\omega_0 t + \theta)} + e^{-j(\omega_0 t + \theta)}}{2} e^{-j\omega t} dt$$

$$= \frac{A_0}{2} e^{j\theta} \int_{-T/2}^{T/2} e^{j(\omega_0 - \omega)t} dt + \frac{A_0}{2} e^{-j\theta} \int_{-T/2}^{T/2} e^{-j(\omega_0 + \omega)t} dt$$

$$= \frac{A_0}{2} e^{j\theta} \left. \frac{e^{j(\omega_0 - \omega)t}}{j(\omega_0 - \omega)} \right|_{-T/2}^{T/2} + \frac{A_0}{2} e^{-j\theta} \left. \frac{e^{-j(\omega_0 + \omega)t}}{-j(\omega_0 + \omega)} \right|_{-T/2}^{T/2}$$

$$= A_0 \left[e^{j\theta} \frac{e^{j(\omega_0 - \omega)T/2} - e^{-j(\omega_0 - \omega)T/2}}{2j(\omega_0 - \omega)} + e^{-j\theta} \frac{e^{j(\omega_0 + \omega)T/2} - e^{-j(\omega_0 + \omega)T/2}}{2j(\omega_0 + \omega)} \right]$$

$$= A_0 e^{j\theta} \frac{\sin(\omega_0 - \omega)T/2}{(\omega_0 - \omega)} + A_0 e^{-j\theta} \frac{\sin(\omega_0 + \omega)T/2}{(\omega_0 + \omega)}$$

$$\text{Let } x_1 = (\omega_0 - \omega)T/2 \text{ and } x_2 = (\omega_0 + \omega)T/2$$

$$= \frac{A_0 T}{2} \left[e^{j\theta} \frac{\sin x_1}{x_1} + e^{-j\theta} \frac{\sin x_2}{x_2} \right]$$

$$\frac{|X_T(f)|^2}{T} = \frac{X_T(f) X_T^*(f)}{T} =$$

$$\left(\frac{A_0 T}{2} \right)^2 \left[e^{j\theta} \frac{\sin x_1}{x_1} + e^{-j\theta} \frac{\sin x_2}{x_2} \right] \left[e^{-j\theta} \frac{\sin x_1}{x_1} + e^{j\theta} \frac{\sin x_2}{x_2} \right]$$

6-40 Cont'd

$$\frac{|X_T(f)|^2}{T} = \frac{A_0^2 T}{4} \left[\left(\frac{\sin x_1}{x_1} \right)^2 + e^{j2\theta} \left(\frac{\sin x_1}{x_1} \right) \left(\frac{\sin x_2}{x_2} \right) + e^{-j2\theta} \left(\frac{\sin x_1}{x_1} \right) \left(\frac{\sin x_2}{x_2} \right) + \left(\frac{\sin x_2}{x_2} \right)^2 \right]$$

Aside:

$$e^{j2\theta} = \int_{-\pi/2}^{\pi/2} e^{j2\theta} \cdot \frac{2}{\pi} d\theta = j^2/\pi$$

$$e^{-j2\theta} = \int_{-\pi/2}^{\pi/2} e^{-j2\theta} \cdot \frac{2}{\pi} d\theta = -j^2/\pi$$

$$\frac{|X_T(f)|^2}{T} = \frac{A_0^2}{4} \left[\frac{T\pi}{\pi} \left(\frac{\sin \pi T(f-f_0)}{\pi T(f-f_0)} \right)^2 + \frac{T\pi}{\pi} \left(\frac{\sin \pi T(f+f_0)}{\pi T(f+f_0)} \right)^2 \right]$$

From Sec. A-8

$$\delta(x) = \lim_{a \rightarrow \infty} \left[\frac{a}{\pi} \left(\frac{\sin ax}{ax} \right)^2 \right]$$

$$\Rightarrow P_x(f) = \frac{A_0^2}{4} \left[\delta(f-f_0) + \delta(f+f_0) \right]$$

6-41 $\overline{x(t)} = A_0 \cos(\omega_0 t + \theta) = A_0 \cdot 0 = 0$

$$R_x(\tau) = \overline{x(t) x(t+\tau)}$$

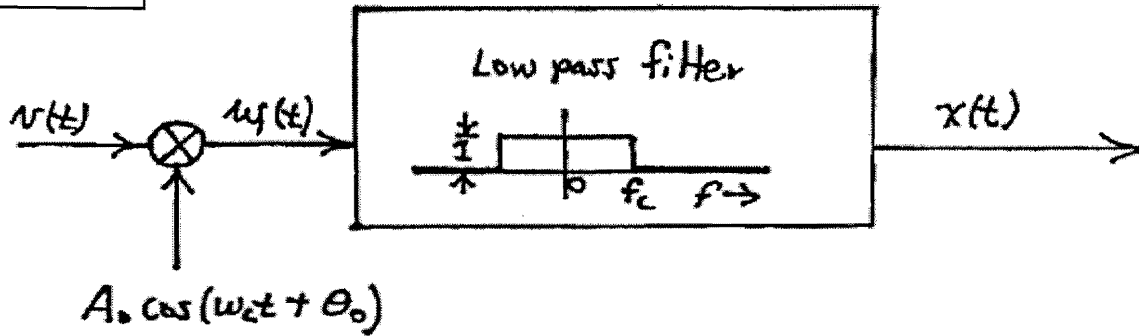
$$= A_0^2 \cos(\omega_0 t + \theta) \cos(\omega_0 t + \omega_0 \tau + \theta)$$

$$= \frac{A_0^2}{2} \cos \omega_0 \tau + \frac{A_0^2}{2} \cos(2\omega_0 t + \omega_0 \tau + 2\theta)$$

$$= \frac{A_0^2}{2} \cos \omega_0 \tau \quad ; \text{ not a function of } t$$

$$\therefore \underline{\underline{x(t) \text{ is W.S.S.}}}$$

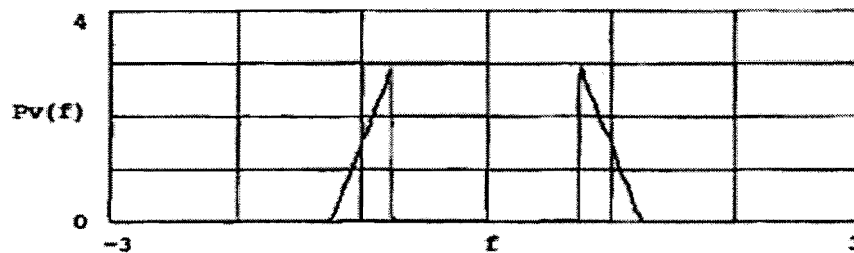
6-43



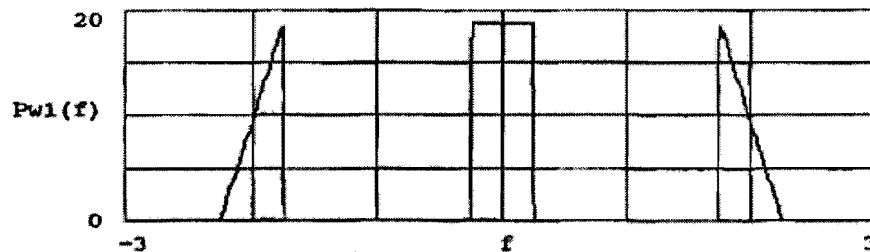
Formulas used in the solution for this problem are derived (essentially) in the textbook. See (6-141) and formulas that follow for $Pw1(f)$ and $Px(f)$ where the 2 in (6-141) is replaced by $(Ao)^2/2$.

$$f := -3, -2.98 \dots 3 \quad Ao := 5 \quad fc := 1$$

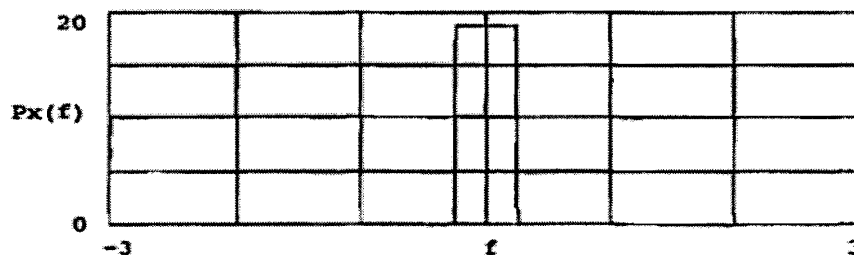
$$Pv(f) := \left[f + \frac{5}{4} \cdot 6 \cdot \left[\frac{1}{2} \left[f + \frac{5}{4} \right] - \frac{1}{2} \left[f + \frac{3}{4} \right] \right] - \left[f - \frac{5}{4} \right] \cdot 6 \cdot \left[\frac{1}{2} \left[f - \frac{3}{4} \right] - \frac{1}{2} \left[f - \frac{5}{4} \right] \right] \right]$$



$$Pw1(f) := \frac{Ao^2}{4} \cdot (Pv(f - fc) + Pv(f + fc))$$



$$Px(f) := \text{if}(|f| < fc, Pw1(f), 0)$$



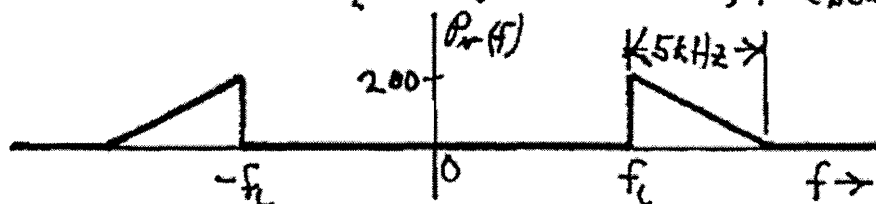
6-45

(a.) $g(t) = 10[x(t) + j\hat{x}(t)]$

$$\Rightarrow G(f) = 10 \left\{ X(f) + j \begin{cases} -jX(f), & f > 0 \\ jX(f), & f < 0 \end{cases} \right\} = \begin{cases} 20X(f), & f > 0 \\ 0, & f < 0 \end{cases}$$

(6-133d) $\Rightarrow P_v(f) = \frac{1}{4} [P_g(f-f_c) + P_g(-f-f_c)] = \frac{1}{4} \begin{cases} 20^2 [P_x(f-f_c) + P_x(f+f_c)], & |f| > f_c \\ 0, & \text{elsewhere} \end{cases}$

Thus, $P_v(f) = 100 \begin{cases} [P_x(f-f_c) + P_x(f+f_c)], & |f| > f_c \\ 0, & \text{elsewhere} \end{cases}$



(b.) The normalized average power is

$$P = \int_{-\infty}^{\infty} P_v(f) df = 2 \left[\frac{1}{2} (200) (5.0k) \right] = 1,000 \text{ Watts}$$

(Area under $P_v(f)$)

$P = 1,000 \text{ Watts (normalized)}$

6-49

$$s(t) = x(t) \cos(\omega_c t + \theta_c)$$

$$-y(t) \sin(\omega_c t + \theta_c)$$

$$= s_{\text{USB}}(t) + s_{\text{LSB}}(t)$$

where

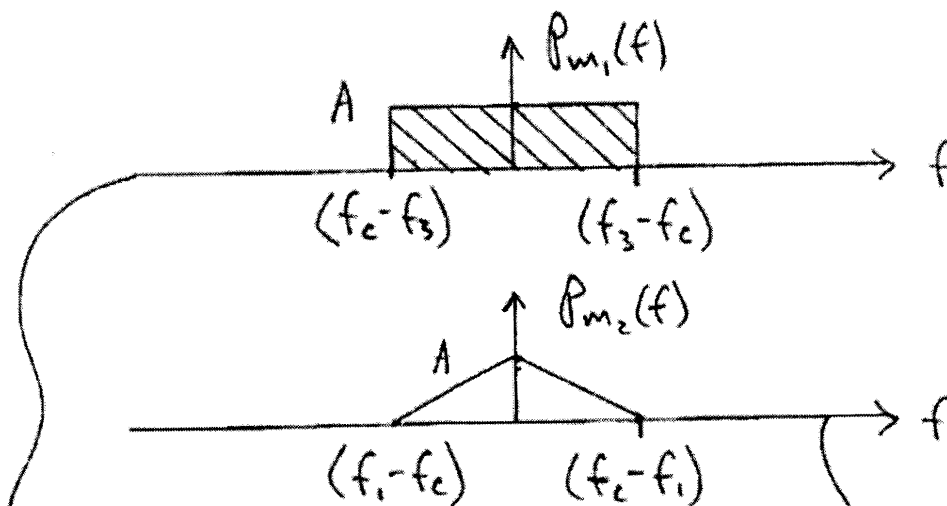
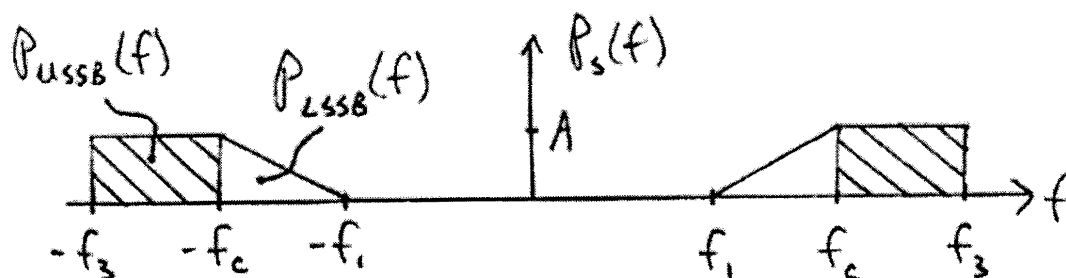
$$s_{\text{USB}}(t) = m_1(t) \cos(\omega_c t + \theta_c) - \hat{m}_1(t) \sin(\omega_c t + \theta_c)$$

$$s_{\text{LSB}}(t) = m_2(t) \cos(\omega_c t + \theta_c) + \hat{m}_2(t) \sin(\omega_c t + \theta_c)$$

6-49 Cont'd

$$\Rightarrow s(t) = [m_1(t) + m_2(t)] \cos(\omega_c t + \theta_c) - [\hat{m}_1(t) - \hat{m}_2(t)] \sin(\omega_c t + \theta_c)$$

$$\Rightarrow \underline{x(t) = m_1(t) + m_2(t)} ; \underline{y(t) = \hat{m}_1(t) - \hat{m}_2(t)}$$



$$\underline{P_{m1}(f) = A \text{ TT} \left(\frac{f}{2(f_3 - f_c)} \right)}$$

$$\underline{P_{m2}(f) = A \wedge \left(\frac{f}{f_c - f_1} \right)}$$

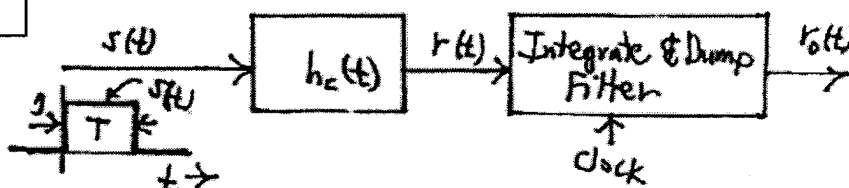
6-52 From example 6-12:

$$P_o(f) = \frac{1}{4} [P_x(f-f_c) + P_x(-f-f_c)]$$

Where, from solution to problem 6-19:

$$P_x(f) = T_b \left[\frac{1 - \cos(\pi f T_b)}{\pi f T_b} \right]^2 = P_x(-f)$$

6-56



$$H_c(f) = \frac{B}{B + jf} = \frac{1}{1 + j\left(\frac{f}{B}\right)}$$

Using Table 2-2

$$\Rightarrow h_c(t) = \begin{cases} 2\pi B e^{-2\pi B t}, & t > 0 \\ 0, & t < 0 \end{cases} \quad \text{Let } a = 2\pi B$$

$$\Rightarrow h_c(t) = \begin{cases} a e^{-at}, & t > 0 \\ 0, & t < 0 \end{cases}$$

$$r(t) = s(t) * h_c(t) = \int_0^t s(\lambda) h_c(t-\lambda) d\lambda = \begin{cases} \int_0^t a e^{-a(t-\lambda)} d\lambda, & 0 < t < T \\ \int_0^T a e^{-a(t-\lambda)} d\lambda, & t > T \end{cases}$$

$$\Rightarrow r(t) = \begin{cases} 1 - e^{-at}, & 0 < t < T \\ e^{-at} [e^{aT} - 1], & t > T \end{cases}$$

$$r_o(t) = \begin{cases} \int_0^t h(\lambda) d\lambda, & 0 < t < T \\ \int_T^t r(\lambda) d\lambda, & T < t < 2T \end{cases} = \begin{cases} \int_0^t [1 - e^{-a\lambda}] d\lambda, & 0 < t < T \\ (e^{aT} - 1) \int_T^t e^{-a\lambda} d\lambda, & T < t < 2T \end{cases}$$

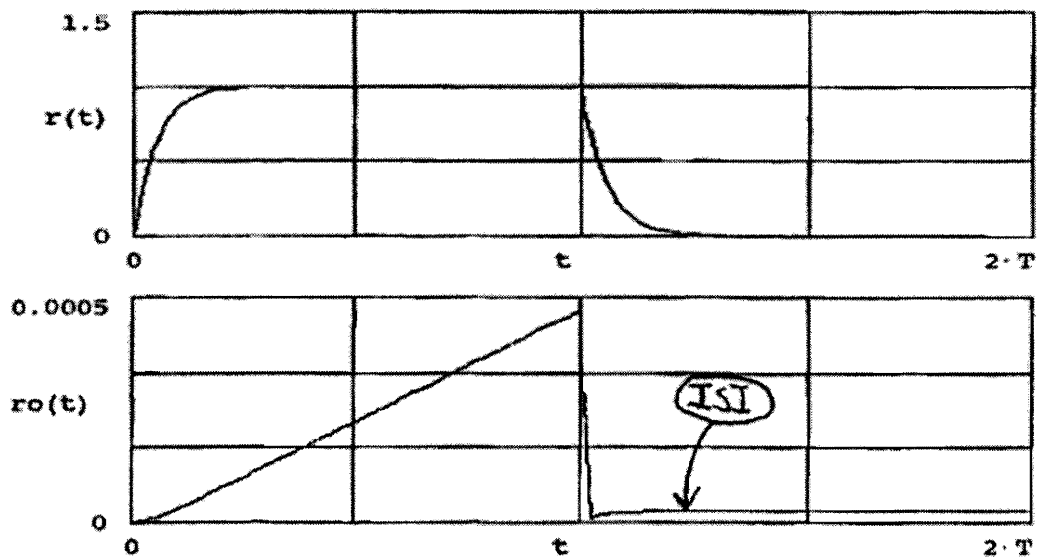
$$\Rightarrow r_o(t) = \begin{cases} t + \frac{1}{a} (e^{-at} - 1), & 0 < t < T \\ \frac{1}{a} (e^{aT} - 1) (e^{-aT} - e^{-at}), & T < t < 2T \end{cases}$$

6-56 Cont'd (a)

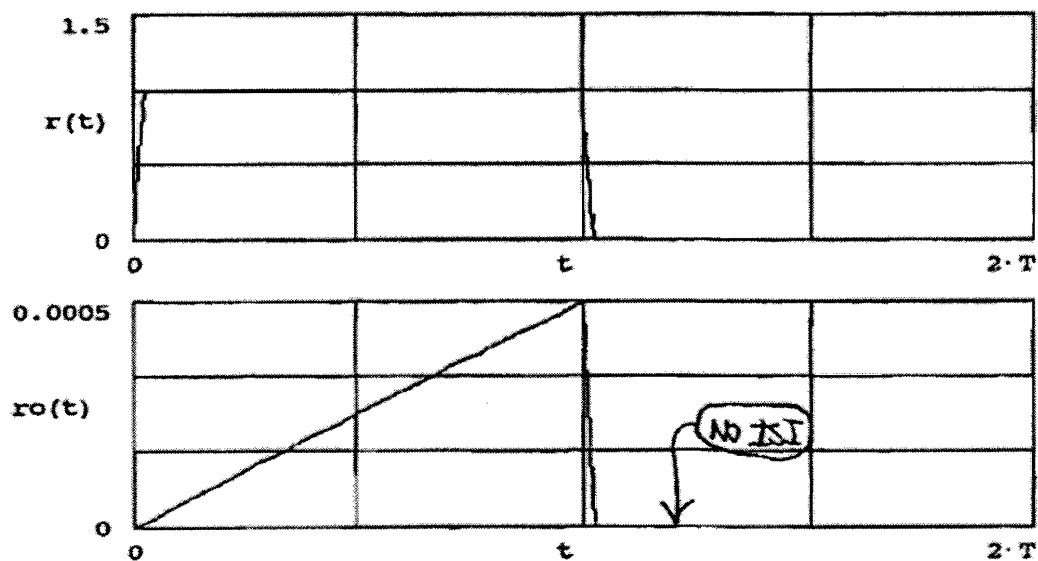
$$R := 2000 \quad T := \frac{1}{R} \quad B := 6000 \quad a := 2 \cdot \pi \cdot B \quad t := 0, \frac{T}{40} \dots 2 \cdot T$$

$$r(t) := \text{if} \left[t \leq T, 1 - e^{-a \cdot t}, [e^{a \cdot T} - 1] \cdot e^{-a \cdot t} \right]$$

$$ro(t) := \text{if} \left[t \leq T, t + \frac{e^{-a \cdot t} - 1}{a}, \frac{e^{a \cdot T} - 1}{a} \cdot [e^{-a \cdot T} - e^{-a \cdot t}] \right]$$



(b.) For all pass channel \nRightarrow Let $B \rightarrow \infty$, \nRightarrow Use $B = 100,000$.
Then, results are as follows.



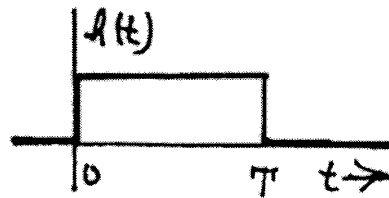
6-58

$$(a.) \quad y(t) = \int_{-\infty}^t [x(t_1) - x(t_1 - T)] dt_1$$

For the impulse response let $x(t) = \delta(t)$.

$$\Rightarrow y(t) = h(t) = \int_{-\infty}^t [\delta(t_1) - \delta(t_1 - T)] dt_1$$

$$h(t) = \begin{cases} \frac{1}{2}, & t < 0 \\ 1, & 0 < t < T \\ \frac{1}{2}, & t = T \\ 0, & t \text{ elsewhere} \end{cases}$$



(b.) $s(t) = h(t_0 - t)$ where $t_0 = T \Rightarrow s(t) = h(t)$ above.

6-59

$$(a.) \quad s(t) = s_1(t) = A \cos \omega_1 t$$

$$w_1(t) = s_1(t) \cos(\omega_1 t) = A \cos^2(\omega_1 t) = \frac{1}{2} A [1 + \cos(2\omega_1 t)]$$

$$N_1(t) = \int_0^t w_1(\lambda) d\lambda = \frac{1}{2} A \int_0^t [1 + \cos(2\omega_1 \lambda)] d\lambda = \frac{1}{2} A \left[\lambda + \frac{\sin(2\omega_1 \lambda)}{2\omega_1} \right]_0^t$$

$$\Rightarrow \underline{N_1(t) = \frac{1}{2} A \left[t + \frac{\sin(2\omega_1 t)}{2\omega_1} \right]}$$

$$w_2(t) = s_1(t) \cos(\omega_2 t) = A \cos(\omega_1 t) \cos(\omega_2 t) \\ = \frac{1}{2} A [\cos(\omega_1 - \omega_2)t + \cos(\omega_1 + \omega_2)t]$$

$$\text{or } w_2(t) = \frac{1}{2} A [\cos(2\Delta\omega t) + \cos((\omega_1 + \omega_2)t)] \text{ where } \Delta\omega = 2\pi\Delta F$$

$$N_2(t) = \int_0^t w_2(\lambda) d\lambda = \frac{1}{2} A \int_0^t [\cos(2\Delta\omega \lambda) + \cos((\omega_1 + \omega_2)\lambda)] d\lambda$$

$$\Rightarrow \underline{N_2(t) = \frac{1}{2} A \left[\frac{\sin(2\Delta\omega t)}{2\Delta\omega} + \frac{\sin((\omega_1 + \omega_2)t)}{\omega_1 + \omega_2} \right]}$$

$$\underline{r_0(t) = N_1(t) - N_2(t)}$$

$$(b.) \quad r(t) = s(t) = A \cos \omega_1 t, \quad 0 < t < T \Rightarrow h(t) = A \cos[\omega_1(T - t)]$$

$$\text{Thus, } r_0(t) = r(t) * h(t) = \int_0^t r(\lambda) h(t - \lambda) d\lambda, \quad 0 < t < T$$

6-59 (b.) Cont'd.

$$\begin{aligned} r_0(t) &= A^2 \int_0^t \cos(\omega_1 \lambda) \cos[\omega_1 (T - (t - \lambda))] d\lambda \\ &= \frac{1}{2} A^2 \int_0^t [\cos(\omega_1 T - \omega_1 t) + \cos(2\omega_1 \lambda + \omega_1 T - \omega_1 t)] d\lambda \\ &= \frac{1}{2} A^2 \left[\lambda \cos(\omega_1 T - \omega_1 t) + \frac{\sin(2\omega_1 \lambda + \omega_1 T - \omega_1 t)}{2\omega_1} \right] \Big|_0^t \\ &= \frac{1}{2} A^2 \left[t \cos(\omega_1 T - \omega_1 t) + \frac{\sin(2\omega_1 t + \omega_1 T - \omega_1 t) - \sin(\omega_1 T - \omega_1 t)}{2\omega_1} \right] \end{aligned}$$

Thus,

$$\underline{r_0(t) = \frac{1}{2} A^2 \left[t \cos \omega_1 (T - t) + \frac{\sin \omega_1 (T + t) - \sin \omega_1 (T - t)}{2\omega_1} \right]}$$

```
f_c := 1000      delta_f := 50      omega1 := 2 * pi * [f_c - delta_f]      omega2 := 2 * pi * [f_c + delta_f]
A := 1           T := 1 / (4 * delta_f)      t := 0, T/100 .. T      delta_omega := 2 * pi * delta_f
```

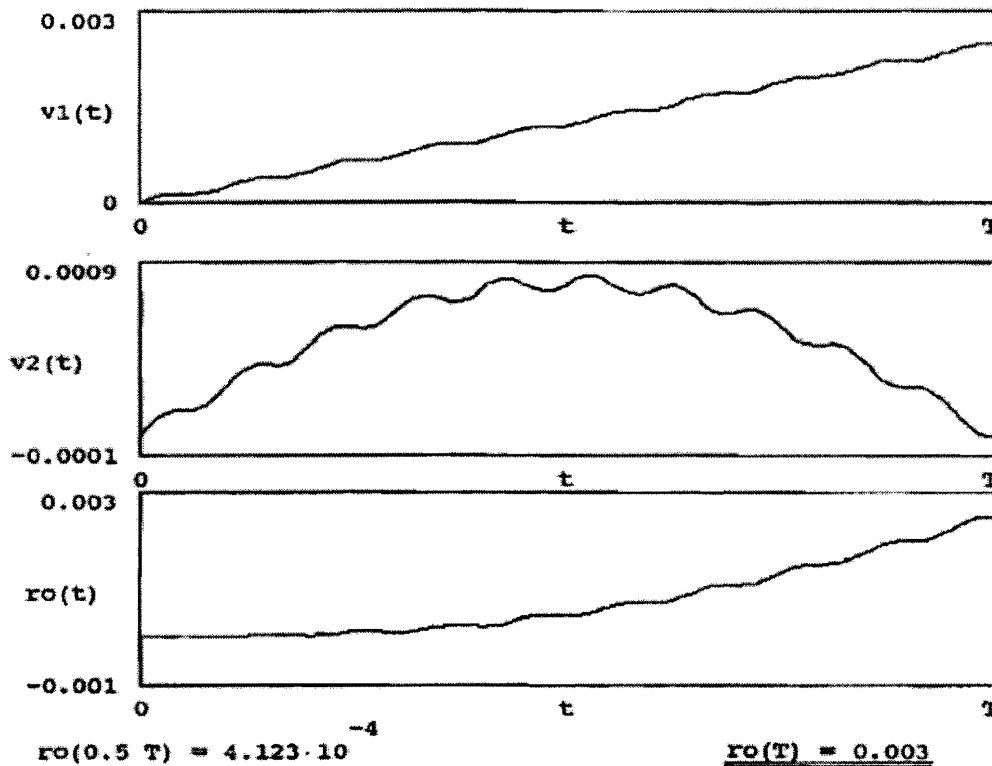
(a.)

$$v_1(t) := \frac{A}{2} \left[t + \frac{\sin(2 \cdot \omega_1 \cdot t)}{2 \cdot \omega_1} \right]$$

$$v_2(t) := \frac{A}{2} \left[\frac{\sin((\omega_1 + \omega_2) \cdot t)}{\omega_1 + \omega_2} + \frac{\sin(2 \cdot \delta \omega \cdot t)}{2 \cdot \delta \omega} \right]$$

$$r_0(t) := v_1(t) - v_2(t)$$

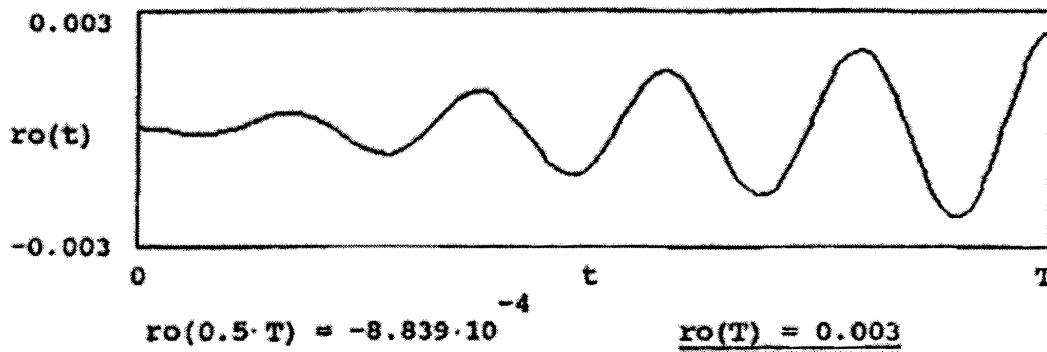
$$T = 0.005$$



6-59 Cont'd.

(b.)

$$r_o(t) := \frac{\lambda^2}{2} \left[t \cdot \cos(\omega_1 (T - t)) + \frac{\sin(\omega_1 (T + t)) - \sin(\omega_1 (T - t))}{2 \omega_1} \right]$$



(c.) The results for parts (a.) and (b.) are different for $0 < t < T$. However, at the sampling time, $t=T$, the results are identical. That is, MathCAD computes $r_o(T)$ to be 0.003 for both cases.